THE ELECTROMAGNETIC INTERACTIONS OF CHARGED FIELDS WITH SPINS ONE AND TWO

A thesis submitted in partial fulfilment of
the requirements for the degree of

DOCTOR OF PHILOSOPHY

by

L.M. NATH, B.Sc. (Hons.), M.Sc. (Dacca)

Tait Institute of Mathematical Physics
University of Edinburgh

June, 1964.

PREFACE

In this dissertation, an attempt has been made to present a systematic study of charged fields with integral spin $s \geq 1$ in interaction with the electromagnetic field. The first part of the work has been devoted mainly to the discussion of the electromagnetic interaction of charged vector mesons with arbitrary magnetic moment. A new method has been proposed to describe these interactions and the theory has been shown to be renormalisable by using the 'ξ-limiting technique'. In the second part, the interaction of charged higher spin fields, in particular the spin 2 field, with the Maxwell field has been studied and it has been found that the local Quantum Field Theory of Bose particles with $s \geq 2$ is inconsistent in the presence of an external electromagnetic field.

All the material presented in this dissertation is asserted to be original except where explicit references are cited. The work described in the first part of this thesis has already been submitted to Nuclear Physics for publication.

It is a pleasure to the author to express his grateful thanks to Professor N. Kemmer, F.R.S. and other members of the department for the hospitality extended to him at the Tait Institute and for the benefit of many useful discussions. Particular thanks are due to Professor N. Kemmer and Dr. P.W. Higgs, who suggested the field of enquiry and whose guidance and encouragement have been of great help throughout the work.

The author would like to acknowledge the award of a Colombo Plan Research Fellowship which has made this work possible.

CONTENTS

C O N T E N T S (Contd.)

....

PART I

ON THE RENORMALIZABLE ELECTROMAGNETIC INTERACTIONS OF CHARGED VECTOR MESONS

PART I

ON THE RENORMALISABLE ELECTROMAGNETIC INTERACTIONS OF CHARGED VECTOR MESONS

1. Introduction

In the conventional renormalisation theory all interactions of vector mesons, except when a neutral vector meson field is coupled with a conserved current, are known to be non-renormalisable. Moreover, in the canonical formalism of the quantised vector meson theory there are the additional difficulties due to the presence of infinite and non-covariant terms in the vacuum expectation value of the T-product of the two field operators. Lee and Yang[1)] have proposed a 'ξ-limiting technique' to circumvent these difficulties associated with the quantised theory of vector mesons. Further, by using this 'ξ-limiting' process and the artifice of a negative metric, the electromagnetic interactions of a charged vector meson with arbitrary magnetic moment have been shown to be renormalisable. However, in their treatment of the anomalous magnetic moment and quadrupole moment of vector mesons which are taken account of in the theory by the introduction of direct interaction of electromagnetic field strengths with the meson field variables, Lee and Yang[1)] have used, it appears from the β-formalism, a particular combination of meson fields and electromagnetic fields.

In the first order formulation of the theory of vector mesons in interaction with the electromagnetic field, which has been discussed here in Section II, there are other interaction terms apart from the ones considered by Lee and Yang. We have shown that these interactions also contribute to the magnetic and quadrupole moment of the charged vector meson, but the presence of these terms in the interaction Lagrangian makes the theory divergent in a non-renormalisable way in the Yang-Lee[1)] scheme. However, it was pointed out by Kemmer[2)] that in the second order formulation one can also describe spin one particles in terms of an antisymmetric tensor of second rank instead of a vector field. In Sections III and IV we have developed at length this 'Inverted scheme' of Kemmer[2)] and discussed the 'ξ-limiting' technique' in this formulation. Although for non-interacting systems the two modes of description are exactly equivalent, they differ considerably when the interaction is introduced. In particular for electromagnetic interactions, the non-renormalisable interaction terms in the Yang-Lee scheme are renormalisable in this formulation (see V).

The Renormalisability of Vector Meson Interactions

1.1. General Criteria of Renormalisability

The theory of an interacting system which is completely determined by the initial interaction Lagrangian and the structure of the causal Green's functions is said to be renormalisable[3)] if

$$\omega_i^{max.} = \frac{1}{2}\sum_{l=1}^{n} (p_l+2) - 4 \leq 0 \qquad (1.1)$$

where

p_l is the highest order of the derivatives in the propagator of the internal line l,

n is the maximum number of the internal lines which can meet at a vertex V_i ,

and ω_i is called the index of the vertex V_i .

If the condition (1.1) is satisfied by an interacting system, then the number of primitively divergent diagrams that can occur in the theory is finite and the introduction of a finite number of counter terms in the Lagrangian removes all the divergences. On the other hand, if $\omega_i^{max.} > 0$, an infinite number of counter terms will be needed to cancel all the divergences and such a theory will be called non-renormalisable.

We shall now discuss specifically the interactions of the vector meson field with the spin $\frac{1}{2}$ Fermion field, the spin 0 boson field and the electromagnetic field.

Let the interaction Lagrangian be of the form

$$\mathcal{L} = \sum_i g_i \mathcal{L}_i (F, V, B, A, D) \qquad (1.2)$$

where we have assumed that the interaction Lagrangian may contain F factors of a Fermion field, V factors of a vector meson field, B factors of a spin 0 Boson field, A factors of the electromagnetic field and D derivatives. Equation (1.1) written in terms of the number of different field factors is

$$\omega_i^{max.} = \tfrac{3}{2}F + 2V + B + A + D - 4 \leq 0. \qquad (1.3)$$

The following are the conventional interaction Lagrangians for the vector meson field in interaction with the Fermion field and the electromagnetic field:

(a) $ig\bar{\Psi}\gamma_\mu\Psi U_\mu$,

(b) $ig\bar{\Psi}\gamma_\mu\gamma_5\Psi U_\mu$,

(c) $ig\bar{\Psi}_e\gamma_\mu(1+\gamma_5)\Psi_\nu U_\mu + ig\bar{\Psi}_\nu\gamma_\mu(1+\gamma_5)\Psi_e U_\mu^+$,

(d) $-\frac{1}{2}ie\left[(\varphi_\nu U_\mu^+ - \varphi_\mu U_\nu^+)(\partial_\nu U_\mu - \partial_\mu U_\nu) - \text{herm. conj.}\right]$
$-\frac{1}{2}e^2(\varphi_\nu U_\mu - \varphi_\mu U_\nu)(\varphi_\nu U_\mu^+ - \varphi_\mu U_\nu^+)$,

(e) $iek F_{\mu\nu}U_\mu^+ U_\nu$,

(f) $ig\bar{\Psi}\sigma_{\mu\nu}\Psi G_{\mu\nu}$,

(g) $g\bar{\Psi}\sigma_{\mu\nu}\gamma_5\Psi G_{\mu\nu}$,

(h) $-ie\varphi_\lambda(\partial_\mu G_{\mu\nu}^+ G_{\lambda\nu} - \partial_\mu G_{\mu\nu} G_{\lambda\nu}^+)$
$+ e^2\varphi_\mu\varphi_\nu G_{\mu\lambda}^+ G_{\nu\lambda}$,

(i) $iek F_{\mu\nu}G_{\mu\rho}^+ G_{\rho\nu}$ (1.4)

where

U_μ is the vector meson field,

φ_μ is the electromagnetic potential,

ψ is a Fermion field with spin $\frac{1}{2}$,

K is a dimensionless constant,

and

$$\sigma_{\mu\nu} = \gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu ,$$

$$F_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu ,$$

$$G_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu ,$$

$$G^+_{\mu\nu} = \partial_\mu U^+_\nu - \partial_\nu U^+_\mu .$$

Now applying the criterion (1.3) one can see that none of the interactions listed in (1.4) is renormalisable. Of course, in these discussions we have not taken account of any fortuitous cancellations that may occur in the theory. In all the interactions in (1.4), it is the longitudinal part of the vector meson propagator which gives rise to the non-renormalisable divergences and in certain cases it may so happen that the individual vertex factors corresponding to the longitudinal part of the vector meson field will cancel one another and the criterion (1.3) will be satisfied by the remaining vertices. The interaction Lagrangian (a) where a neutral vector meson field has been coupled to a conserved current is one such example. This Lagrangian does not satisfy the renormalisability criterion (1.3). So it is apparently a non-renormalisable interaction, but in actual fact the non-renormalisable divergences do not occur in this theory. In order to show this we shall introduce the Stueckelberg formalism of the vector meson theory.

The vector meson field $U_\mu(x)$ is decomposed into a transverse and a longitudinal part

$$U_\mu(x) = B_\mu(x) + \frac{1}{m}\partial_\mu B(x) \tag{1.5}$$

where the fields $B_\mu(x)$ and $B(x)$ satisfy the following equation of motion and the commutation relations in the interaction picture:

$$(\Box - m^2) B_\mu(x) = 0 \ , \tag{1.6}$$

$$(\Box - m^2) B(x) = 0 \ , \tag{1.7}$$

$$[B_\mu(x), B_\nu(y)] = \delta_{\mu\nu}\Delta(x-y) \ , \tag{1.8}$$

$$[B(x), B(y)] = \Delta(x-y) \tag{1.9}$$

In the interaction picture the state vector $\Psi(t)$ satisfies the following Schroedinger equation[4]

$$i\frac{\partial \Psi(t)}{\partial t} = \int H_1(x)\, d^3x \ \Psi(t) \tag{1.10}$$

together with the subsidiary condition

$$[\partial_\mu B_\mu(x) + m B(x)]\Psi(t) = 0 \tag{1.11}$$

where

$$H_1(x) = -ig\overline{\psi}(x)\gamma_\mu\psi(x)U_\mu(x) - \frac{1}{2m^2}j_4^2(x) \ ,$$
$$j_4(x) = ig\overline{\psi}(x)\gamma_4\psi(x) \tag{1.12}$$

and $d^3x = dx_1 dx_2 dx_3$.

Now we transform the system with the unitary operator so that

$$\Psi(t) \longrightarrow \Psi'(t) = U(t)\,\Psi(t)\,, \tag{1.13}$$

$$O(t) \longrightarrow O'(t) = U(t)\,O(t)\,U^{-1}(t) \tag{1.14}$$

where

$O(t)$ is any operator in the interaction picture and

$U(t)$ is given by

$$U(t) = e^{-ig/m\int \overline{\Psi}(x)\gamma_4\Psi(x)B(x)\,d^3x} \quad . \tag{1.15}$$

In the new representation equation (1.10) is given by

$$i\frac{\partial\Psi'(t)}{\partial t} = -ig\int\overline{\Psi}(x)\gamma_\mu\Psi(x)B_\mu(x)\,d^3x \;\; \Psi'(t) \tag{1.16}$$

and the subsidiary condition is

$$\left[\partial_\mu B_\mu(x) + mB(x) + ig\int j_4(x')\,\Delta(x-x')\,d^3x'\right]\Psi'(t) = 0 \quad . \tag{1.17}$$

One can see from equation (1.16) that the interaction Hamiltonian containing the longitudinal part of the vector meson field disappears in the new representation and so does the surface term $-\frac{1}{2}m^{-2}j_4^{\,2}$. The new interaction Hamiltonian in equation (1.16) satisfies the renormalisability criterion (1.3). The scattering matrix S' in the new representation is related to the scattering matrix S in the former representation by the following equation :

$$S' = U(\infty)\,S\,U(-\infty) \tag{1.18}$$

Assuming the adiabatic hypothesis under which the interaction is switched off at time $t = \pm\infty$, we get

$$U(+\infty) = U^{-1}(-\infty) = 1$$

$$\text{and } S' = S \tag{1.19}$$

so the interaction (a) in (1.4) is really renormalisable.

Regarding the renormalisability of the interaction Lagrangian (b) in (1.3), where a neutral vector meson field has been coupled to a partially conserved current, there have been controversial claims by different authors. Glashow[5] and Divakaran[6] have claimed that this interaction is renormalisable On the other hand, Salam, Kamefuchi and O'Raifeartaigh have shown it to be non-renormalisable. In what follows we shall make a brief review of the two methods and try to resolve the controversy.

The current j_μ^5 given by

$$j_\mu^5 = ig\bar{\psi}\gamma_\mu\gamma_5\psi \tag{1.20}$$

is not conserved, but satisfies what is called a partial conservation law:

$$\partial_\mu j_\mu^5 = 2iMg\bar{\psi}\gamma_5\psi \tag{1.21}$$

where M is the mass of the Fermion field. Equation (1.21) is true in the Heisenberg picture also. By making use of the Stueckelberg formalism, the interaction Lagrangian (b) can be written as

$$ig\bar{\psi}\gamma_\mu\gamma_5\psi U_\mu = ig\bar{\psi}\gamma_\mu\gamma_5\psi B_\mu + igm^{-1}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu B \tag{1.22}$$

Now if we neglect a four divergence and use equation (1.21), the interaction Lagrangian reduces to the following:

$$ig\bar{\Psi}\gamma_\mu\gamma_5\Psi U_\mu = ig\bar{\Psi}\gamma_\mu\gamma_5\Psi B_\mu - 2igMm^{-1}\bar{\Psi}\gamma_5\Psi B \qquad (1.23)$$

The commutation relations in the interaction picture for the Stueckelberg fields are given by equations (1.8) and (1.9), and those for the Fermi fields are

$$\left[\psi_\alpha(x), \bar{\psi}_\beta(x')\right] = -iS_{\alpha\beta}(x-x') \qquad (1.24)$$

where $S_{\alpha\beta}(x-x') = (\gamma_\mu\partial_\mu - m)\,\Delta(x-x')$

so, both the terms in (1.23) satisfy the renormalisability criteria with $\omega_i^{max.} = 0$. This is the approach used by Glashow[5)] and Divakaran[6)] to show the renormalisability of a partially conserved current coupled to a neutral vector meson field.

Salam, Kamefuchi and O'Raifeartaigh[7)] have rigorously established and made use of an equivalence theorem to show the non-renormalisability of the interaction (b) in (1.3). This equivalence theorem which was first proved by Chisholm states that if the field variables of an interacting system in Quantum Field Theory undergo a point transformation represented by

$$\varphi_r(x) \longrightarrow \varphi_r'(x) = \varphi_r'\left\{\varphi_s(x)\right\} \qquad (1.25)$$

where $\varphi_r'(x)$ contain the variables $\varphi_s(x)$ but not their time derivatives and also $\varphi_r(x)$ can be expressed as a power series in terms of the variables $\varphi_s(x)$ and vice versa, then the Lagrange equations, the canonical commutation relations, the energy-momentum tensor $T_{\mu\nu}$ and the S-matrix remain invariant under this transformation.

Let the Lagrangian density for the vector meson field interacting with the Fermi field be given by

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{B_\mu} + \mathcal{L}_B + \mathcal{L}_1 \tag{1.26}$$

where

$$\mathcal{L}_F = \bar{\Psi}(\gamma_\mu \partial_\mu + M)\Psi \quad ,$$

$$\mathcal{L}_{B_\mu} = -\tfrac{1}{2}\partial_\lambda B_\mu \partial_\lambda B_\mu - \tfrac{1}{2} m^2 B_\lambda B_\lambda \, ,$$

$$\mathcal{L}_B = -\tfrac{1}{2}\partial_\lambda B \, \partial_\lambda B - \tfrac{1}{2} m^2 B \cdot B \quad ,$$

$$\mathcal{L}_1 = i g \bar{\Psi} \gamma_\mu \gamma_5 \Psi \, U_\mu$$

and $\quad U_\mu = B_\mu + \frac{1}{m}\partial_\mu B$

After applying the point transformation

$$\begin{aligned}
\psi(x) &\longrightarrow \psi'(x) = e^{-ig/m \, \gamma_5 B(x)} \psi(x) \, , \\
\bar{\Psi}(x) &\longrightarrow \bar{\Psi}'(x) = \bar{\Psi}(x) \, e^{-ig/m \, \gamma_5 B(x)} \, , \\
B_\mu(x) &\longrightarrow B'_\mu(x) = B_\mu(x) \, , \\
B(x) &\longrightarrow B'(x) = B(x)
\end{aligned} \tag{1.27}$$

and expressing the transformed variables in terms of the original ones we get

$$\mathcal{L}(\psi', \bar{\psi}'; B'_\mu, B'_{\mu,\lambda}; B', B'_{,\lambda})$$
$$= \bar{\psi}(\gamma_\mu\partial_\mu + M)\psi - \tfrac{1}{2}\partial_\lambda B_\mu \partial_\lambda B_\mu - \tfrac{1}{2}\partial_\lambda B\,\partial_\lambda B$$
$$-\tfrac{1}{2}m^2 B_\lambda B_\lambda - \tfrac{1}{2}m^2 B\cdot B + ig\bar{\psi}\gamma_\mu\gamma_5\psi B_\mu$$
$$+\bar{\psi}\left(e^{-\frac{2ig}{m}\gamma_5 B} - 1\right)\psi \tag{1.28}$$

The interaction Lagrangian is now given by

$$\mathcal{L}_1 = ig\bar{\psi}\gamma_\mu\gamma_5\psi B_\mu + \bar{\psi}\left(e^{-\frac{2ig}{m}\gamma_5 B} - 1\right)\psi \tag{1.29}$$

The first term in the expression (1.29) is renormalisable. The second term gives an infinite number of primitively divergent diagrams and there is no possibility of any cancellation in the higher orders. The same result can also be derived by using other methods[9), 10)] and one of them is to apply the following unitary transformation

$$\Psi(t) \rightarrow \Psi'(t) = U(t)\,\Psi(t)$$
$$O(t) \rightarrow O'(t) = U(t)\,O(t)\,U^{-1}(t) \tag{1.30}$$

where

$\Psi(t)$ is the state vector,

$O(t)$ is any operator in the interaction picture,

and $U(t)$ is given by

$$U(t) = e^{-\frac{ig}{m}\int \bar{\psi}(x)\gamma_4\gamma_5\psi(x)B(x)\,d^3x} \tag{1.31}$$

In our opinion the interaction Lagrangian $ig\bar{\Psi}\gamma_\mu\gamma_5\Psi U_\mu$ is non-renormalisable and the result claimed by Glashow and Divakaran is erroneous. They have shown the equivalence between the interaction Lagrangians (1.22) and (1.23) by using the equation (1.21) which is a consequence of the equation of motion of the Fermion field and the use of this property of the current j_μ^5 in the Lagrangian itself has produced the erroneous result derived by them.

The minimal electromagnetic interaction of the charged vector meson field represented by the interaction Lagrangian (d) in (1.4) is non-renormalisable according to the conventional renormalisability criterion (1.3). Yang and Lee[1)] have developed a method called the ' ξ -limiting process' and by using this technique they have shown that the electromagnetic interaction of a charged vector meson with arbitrary magnetic moment is renormalisable. In fact, by using the ' ξ -limiting process' all the interactions from (a) to (e) listed in (1.4) can be shown to be renormalisable. This method consists essentially in introducing in the Lagrangian an additional term dependent on the dimensionless parameter ' ξ ' which in the limit tends to zero. As a result of this the vector field U_μ describes, in the absence of interactions, not only the spin one particles but a system of uncoupled spin one and spin zero mesons. The vacuum expectation value of the T -product of two operators U_μ and U_μ^+ contains two parts: one corresponds to the propagation of the spin one particles and the other corresponds to the propagation of the scalar mesons. At high energies the spin zero part of the propagator behaves like a regulator to the longitudinal part of the vector meson propagator and the theory becomes renormalisable provided the limit $\xi \to 0$ exists.

In the usual canonical formalism where the vector field U_μ describes the spin one particles, it is not possible to renormalise the interactions (f), (g), (h) and (i) in (1.3) even by using the ξ-limiting technique. In the inverted scheme which we have developed here these interactions have been shown to be renormalisable with the aid of the ξ-limiting technique in this inverted scheme.

2. Electromagnetic Interaction of Vector Mesons with Anomalous Magnetic Moment.

The equations of motion for spin one particles can be written in a variety of ways. But as the second order equations can always be derived from the first order equations and as also the definition of a minimal electromagnetic interaction is not unique in the second order formulation, we prefer to work with the first order form of the theory, in particular, with the Kemmer form[11)] of the equations.

The equations of motion in this formalism are:

$$(\beta_\mu \partial_\mu + m)\psi = 0 \tag{2.1}$$

where ψ is a column matrix of ten components and the β_μ are (10 x 10) matrices obeying the following commutation rule:

$$\beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\nu \beta_\mu = \beta_\mu \delta_{\nu\lambda} + \beta_\lambda \delta_{\nu\mu} . \tag{2.2}$$

Four of the ten equations following from equation (2.1) are not, strictly speaking, equations of motion. They do not contain time derivatives in any representation for the β_μ and can be regarded as subsidiary conditions. The remaining six equations correspond to the six degrees of freedom necessary to describe a particle with spin one. Equations (2.1) can be derived from the following Lagrangian with the help of a variational principle

$$\mathcal{L} = \overline{\Psi}(\partial_\mu \beta_\mu + m)\Psi \tag{2.3}$$

The Minimal electromagnetic interaction is introduced by the gauge invariant replacement, $\partial_\mu \rightarrow \partial_\mu - ie\varphi_\mu$ where φ_μ is the electromagnetic potential.

After such a replacement equation (2.1) describes a vector meson with magnetic moment $e/2m$ and no quadrupole moment. If translated into vector notation, the interaction Lagrangian and the equations of motion will have the following form :

$$\mathcal{L}_1 = \tfrac{1}{2} ie\, G^{+}_{\mu\nu} g_{\mu\nu} - \tfrac{1}{2} ie\, G_{\mu\nu} g^{+}_{\mu\nu} \quad , \tag{2.4}$$

$$\begin{aligned} m G_{\mu\nu} &= \partial^{-}_{\mu} A_\nu - \partial^{-}_{\nu} A_\mu \quad , \\ \partial^{-}_{\mu} G_{\mu\nu} &= m A_\nu \quad ; \end{aligned} \tag{2.5}$$

$$\begin{aligned} G_{\mu\nu} &= \Gamma_{\mu\nu}\Psi \quad , \quad A_\mu = \Gamma_\mu \Psi \quad , \\ \beta_\mu &= \tilde{\Gamma}_\nu \Gamma_{\mu\nu} + \tilde{\Gamma}_{\mu\nu}\Gamma_\nu \quad . \end{aligned} \tag{2.6}$$

The quantities Γ_μ are the Harisch Chandra matrices and

$$g_{\mu\nu} = \varphi_\mu A_\nu - \varphi_\nu A_\mu \,, \quad g^{+}_{\mu\nu} = \varphi_\mu A^{+}_\nu - \varphi_\nu A^{+}_\mu \,,$$
$$m G^{+}_{\mu\nu} = \partial^{+}_{\mu} A^{+}_\nu - \partial^{+}_{\nu} A^{+}_\mu \,, \quad \partial^{-}_{\mu} = \partial_\mu - ie\varphi_\mu \,, \quad \partial^{+}_{\mu} = \partial_\mu + ie\varphi_\mu \,,$$

$+$ = complex conjugate times $(-1)^N$, where N = number of subscripts 4

The anomalous magnetic moment of the particle, if it has any, can be accounted for by introducing the term $ie\kappa\bar{\Psi}\beta_\mu\beta_\nu\Psi F_{\mu\nu}$ in the Lagrangian where κ is a constant. By using the expressions (2.6) we get

$$ie\kappa\bar{\Psi}\beta_\mu\beta_\nu\Psi F_{\mu\nu} = ie\kappa F_{\mu\nu}A_\mu^+ A_\nu + ie\kappa F_{\mu\nu}G_{\mu\rho}^+ G_{\rho\nu} \tag{2.7}$$

It was first pointed out by Pauli[12] that an interaction term of the form $ie\kappa F_{\mu\nu}A_\mu^+ A_\nu$ in the Lagrangian will contribute to the magnetic moment of the particle. In the Appendix we have shown that the second term in expression (2.7) will also add to the magnetic moment and the quadrupole moment of vector mesons. For a positive vector meson they are, respectively,

$$\begin{aligned} \underset{\sim}{M} &= -e\kappa/2m\, \underset{\sim}{S}, \\ Q &= e\kappa/2m^2 \end{aligned} \tag{2.8}$$

where S is the spin of the vector meson.

Lee and Yang[1] have considered only the first term in expression (2.7) to take into account the anomalous magnetic moment of the particle. Inclusion of an interaction term like $ie\kappa F_{\mu\nu}G_{\mu\rho}^+ G_{\rho\nu}$ will make the theory non-renormalisable in their formalism. But from the β-formalism, it is clear that the presence of the second term of ecpression (2.7) in the interaction Lagrangian is as much justifiable as that of the first term. In fact, our way of introducing the direct interaction of E - M field strengths with the vector meson field is the most natural one, as $(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu)$ is the spin operator in the first order formulation of the theory.

Of course, if instead of using $\frac{1}{2}ie\kappa\bar{\Psi}(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu)\Psi F_{\mu\nu}$ one uses $\frac{1}{2}ie\kappa\bar{\Psi}(1-\varkappa B)(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu)\Psi F_{\mu\nu}$ in the interaction Lagrangian, where $B = \beta_\lambda\beta_\lambda$, then

for $\varkappa = \frac{1}{2}$

$$\begin{aligned}&\tfrac{1}{2}ie\kappa\bar{\Psi}(1-\varkappa B)(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu)\Psi F_{\mu\nu}\\ &\qquad = \tfrac{1}{2}ie\kappa F_{\mu\nu}A_\mu^+ A_\nu\end{aligned} \tag{2.9}$$

and for $\varkappa = \frac{1}{3}$

$$\begin{aligned}&\tfrac{1}{2}ie\kappa\bar{\Psi}(1-\varkappa B)(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu)\Psi F_{\mu\nu}\\ &\qquad = \tfrac{1}{3}ie\kappa F_{\mu\nu}G_{\mu\rho}^+ G_{\rho\nu}\end{aligned} \tag{2.10}$$

For any other value of $\varkappa$, we have on the right hand side a linear combination of expressions (2.9) and (2.10).

The equations of motion in first order which correspond to $\varkappa = \frac{1}{2}$ and $\frac{1}{3}$ are given, respectively, by

$$\begin{aligned}m G_{\mu\nu} &= \bar{\partial_\mu} A_\nu - \bar{\partial_\nu} A_\mu\\ \bar{\partial_\mu} G_{\mu\nu} &= m A_\nu + \tfrac{1}{2}ie\kappa F_{\mu\nu}A_\mu\end{aligned} \tag{2.11}$$

and

$$\begin{aligned}m G_{\mu\nu} &= \bar{\partial_\mu} A_\nu - \bar{\partial_\nu} A_\mu + \tfrac{1}{3}ie\kappa(F_{\mu\lambda}G_{\lambda\nu} - F_{\nu\lambda}G_{\lambda\mu})\\ \bar{\partial_\mu} G_{\mu\nu} &= m A_\nu\end{aligned} \tag{2.12}$$

The second order equations following from equations (2.11) are

$$\partial_{\bar\mu}\left(\partial_{\bar\mu} A_\nu - \partial_{\bar\nu} A_\mu\right) - m^2 A_\nu - ie\kappa_1 F_{\mu\nu} A_\mu = 0 \tag{2.13}$$

where $\kappa_1 = \frac{1}{2}\kappa m$.

To get second order equations from equations (2.12), it is more convenient to eliminate A_μ and write down the equations in terms of $G_{\mu\nu}$, Thus we get

$$\begin{aligned} &\partial_{\bar\mu}\left(\partial_{\bar\lambda} G_{\lambda\nu}\right) - \partial_{\bar\nu}\left(\partial_{\bar\lambda} G_{\lambda\mu}\right) - m^2 G_{\mu\nu} \\ &\quad + ie\kappa_2\left(F_{\mu\lambda} G_{\lambda\nu} - F_{\nu\lambda} G_{\lambda\mu}\right) = 0 \end{aligned} \tag{2.14}$$

where $\kappa_2 = \frac{1}{3}\kappa m$

Equation (2.13) is the conventional form of the wave equation for a charged vector meson in interaction with the electromagnetic field and having magnetic moment $\frac{1}{2}(1-\kappa_1)\, e/m$ and quadrupole moment $e\kappa_1/m^2$. Equation (2.14) also describes a vector meson with magnetic moment $\frac{1}{2}(1-\kappa_2)\, e/m$ and quadrupole moment $e\kappa_2/m^2$. This scheme, though comparatively unfamiliar, has the advantage that one can consider an interaction term like $ie\kappa F_{\mu\nu} G^{+}_{\mu\rho} G_{\rho\nu}$ in this formalism. In the following sections we discuss the -limiting technique in this inverted scheme.

3.1. Lagrangian

We shall discuss first the free vector meson field in the inverted scheme. Equation (2.14) with $e = 0$ can be obtained from the following Lagrangian:

$$\mathcal{L} = \partial_\mu G^{+}_{\mu\nu} \partial_\lambda G_{\lambda\nu} + \tfrac{1}{2} m^2 G^{+}_{\mu\nu} G_{\mu\nu} \tag{3.1}$$

The free Hamiltonian H_o is given by

$$H_0 = -\underset{\sim}{\Pi}_4^{+} \cdot \underset{\sim}{\Pi}_4 - \tfrac{1}{m^2} (\underset{\sim}{\nabla} \times \underset{\sim}{\Pi}_4^{+}) (\underset{\sim}{\nabla} \times \underset{\sim}{\Pi}_4) - (\underset{\sim}{\nabla} \cdot \underset{\sim}{G}_4^{+}) (\underset{\sim}{\nabla} \cdot \underset{\sim}{G}_4) - m^2 \underset{\sim}{G}_4^{+} \cdot \underset{\sim}{G}_4 \tag{3.2}$$

where $\underset{\sim}{\Pi}_4 \equiv \Pi_{4k}$, $\underset{\sim}{G}_4 \equiv G_{4k}$; $k = 1, 2, 3$.

Π_{4k} and Π^{+}_{4k} are respectively the momenta canonically conjugate to G_{4k} and G^{+}_{4k}:

$$\begin{gathered} \Pi_{4k} = -i \partial_\lambda G^{+}_{\lambda k}, \quad \Pi^{+}_{4k} = -i \partial_\lambda G_{\lambda k} ; \\ \Pi^{+}_{4k} = -\Pi^{*}_{4k}, \quad G^{+}_{4k} = -G^{*}_{4k} . \end{gathered} \tag{3.3}$$

The G_{ik} are not independent 'coordinates' but can be expressed in terms of other canonically independent variables. They are given explicitly by

$$\begin{gathered} G_{ik} = i m^{-2} (\underset{\sim}{\nabla} \times \underset{\sim}{\Pi}_4^{+})_{ik}, \quad G^{+}_{ik} = i m^{-2} (\underset{\sim}{\nabla} \times \Pi_4)_{ik} ; \\ G^{+}_{ik} = G^{*}_{ik} . \end{gathered} \tag{3.4}$$

The free field can be expressed in terms of the creation and annihilation operators of the transverse and longitudinal mesons. In terms of these operators one has the following representation for the fields and their conjugate momenta:

$$\underset{\sim}{G}_4 = \sum_{\underset{\sim}{k},r} (2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{(r)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) + b_{\underset{\sim}{k}}^{*(r)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t)\right] (\omega/m)\, \underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+ \sum_{\underset{\sim}{k}} (2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{(3)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) + b_{\underset{\sim}{k}}^{*(3)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t)\right] \hat{\underset{\sim}{k}} \;,$$

$$\underset{\sim}{\Pi}_4 = \sum_{\underset{\sim}{k},r} i(2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{*(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) - b_{\underset{\sim}{k}}^{(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t)\right] m\, \underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+ \sum_{\underset{\sim}{k}} i(2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{*(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) - b_{\underset{\sim}{k}}^{(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t)\right] \omega\, \hat{\underset{\sim}{k}} \qquad (3.5)$$

where $\underset{\sim}{e}_{\underset{\sim}{k}}^{(1)}$, $\underset{\sim}{e}_{\underset{\sim}{k}}^{(2)}$ and $\hat{\underset{\sim}{k}} = \underset{\sim}{k}|\underset{\sim}{k}|^{-1}$ form a right-handed orthonormal set of unit vectors, $\omega = (\underset{\sim}{k}^2 + m^2)^{\frac{1}{2}} > 0$ and Ω = normalisation volume.

In the expressions (2.5) $a_{\underset{\sim}{k}}^{(r)}$ (r = 1, 2) and $a_{\underset{\sim}{k}}^{(3)}$ are the annihilation operators for the positively charged transverse and longitudinal mesons, respectively. $a_{\underset{\sim}{k}}^{*(r)}$ and $a_{\underset{\sim}{k}}^{*(3)}$ are the corresponding creation operators. Similarly, $b_{\underset{\sim}{k}}$ and $b_{\underset{\sim}{k}}^{*}$ are the annihilation and creation operators for the negatively charged mesons. In terms of these operators the free Hamiltonian H_0 is given by

$$H_0 = \sum_{\underset{\sim}{k}, r} \omega \left(a^{*(r)}_{\underset{\sim}{k}} a^{(r)}_{\underset{\sim}{k}} + \tfrac{1}{2} \right) + \sum_{\underset{\sim}{k}} \omega \left(a^{*(3)}_{\underset{\sim}{k}} a^{(3)}_{\underset{\sim}{k}} + \tfrac{1}{2} \right) \tag{3.6}$$

$$+ \text{ same terms with } a \longrightarrow b \quad .$$

The commutation relations at equal time are given by

$$\begin{aligned} \left[\Pi_{4k}(\underset{\sim}{r}, t),\, G_{4l}(\underset{\sim}{r}', t) \right] &= -i \delta_{kl} \delta^3(\underset{\sim}{r} - \underset{\sim}{r}') , \\ \left[\Pi^{+}_{4k}(\underset{\sim}{r}, t),\, G^{+}_{4l}(\underset{\sim}{r}', t) \right] &= -i \delta_{kl} \delta^3(\underset{\sim}{r} - \underset{\sim}{r}') \end{aligned} \tag{3.7}$$

and all other commutators between $G_{4k}, G^{+}_{4k}, \Pi_{4k}, \Pi^{+}_{4k}$ are zero.

The covariant commutation relations are

$$\begin{aligned} &\left[G_{\mu\nu}(x),\, G^{+}_{\lambda\rho}(x') \right] \\ &= -m^{-2} \left[\delta_{\mu\lambda} \partial_\nu \partial_\rho + \delta_{\nu\rho} \partial_\mu \partial_\lambda - \delta_{\mu\rho} \partial_\nu \partial_\lambda \right. \\ &\qquad \left. - \delta_{\nu\lambda} \partial_\mu \partial_\rho \right] \Delta(x - x') \end{aligned} \tag{3.8}$$

where

$$\Delta(x - x') = -i \, (8\pi^3)^{-1} \int \exp ik(x - x') \, \delta(k^2 + m^2) \, \epsilon(k^0) \, d^4k$$

3.2. Interaction and Feynman Diagrams

The minimal electromagnetic interaction can be introduced by the gauge invariant replacement, $\partial_\mu \rightarrow \partial_\mu - ie\varphi_\mu$, in the Lagrangian (2.1). Then by a unitary transformation we go over to the interaction representation. In this picture the interaction Hamiltonian and the propagator of the vector meson field are given, respectively, by

$$
\begin{aligned}
H_{int} &= ie\varphi_\lambda\left(\partial_\mu G^{+}_{\mu\nu} G_{\lambda\nu} - \partial_\mu G_{\mu\nu} G^{+}_{\lambda\nu}\right) \\
&\quad - e^2 \varphi_\mu \varphi_\nu G^{+}_{4\mu} G_{4\nu} - e^2 m^{-2} \left(\underset{\sim}{\varphi} \times \underset{\sim}{\Pi}_4^{+}\right)\left(\underset{\sim}{\varphi} \times \underset{\sim}{\Pi}_4\right)
\end{aligned}
\tag{3.9}
$$

where $\Pi_{4k} = -i\partial_\lambda G^{+}_{\lambda k}$, $\Pi^{+}_{4k} = -i\partial_\lambda G_{\lambda k}$

where $\underset{\sim}{\Pi}_4 \equiv \Pi_{4k}$, $k = 1, 2, 3$;

$$
\begin{aligned}
&\left\langle T\left[G_{\mu\nu}(x)\, G^{+}_{\lambda\rho}(y)\right]\right\rangle_{vac.} \\
&= -m^{-2}\left[\delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\rho}\partial_\mu\partial_\lambda - \delta_{\mu\rho}\partial_\lambda\partial_\nu - \delta_{\nu\lambda}\partial_\mu\partial_\rho\right]\Delta_F(x-y) \\
&\quad + i m^{-2}\left[\delta_{\nu\rho}\delta_{\mu 4}\delta_{\lambda 4} + \delta_{\mu\lambda}\delta_{\nu 4}\delta_{\rho 4} - \delta_{\nu\lambda}\delta_{\mu 4}\delta_{\rho 4} - \delta_{\mu\rho}\delta_{\nu 4}\delta_{\lambda 4}\right]\delta^4(x-y)
\end{aligned}
\tag{3.10}
$$

where

$$
\Delta_F(x-y) = -i(2\pi)^{-4}\int d^4p\, e^{ip(x-y)}\,(p^2+m^2-i\epsilon)^{-1}
$$

$$
d^4p = d^3\underset{\sim}{p}\,(-i\,dp_4)\ , \quad p^2 = p_\lambda p_\lambda
$$

In the canonical formalism the field variables G_{ik} are not 'independent coordinates'. They are treated as dependent coordinates in terms of $\underset{\sim}{G}_4$ and the momenta conjugate to $\underset{\sim}{G}_4$. So the interaction Hamiltonian has a non-covariant term, and the non-covariant terms in the propagator are needed for the continuity of the T -product of the field variables at time t = 0. But according to a general theorem proved in reference 1) the entire S-matrix calculated from $H_{int.}$ (3.9) by using the expression (3.10) as the vector meson propagator is the same as that generated by the following modified interaction Hamiltonian and modified propagator:

$$H_{int} = ie\varphi_\lambda(\partial_\mu G^+_{\mu\nu} G_{\lambda\nu} - \partial_\mu G_{\mu\nu} G^+_{\lambda\nu})$$
$$- e^2 \varphi_\mu \varphi_\nu G^+_{\mu\lambda} G_{\nu\lambda} \quad , \qquad (3.11)$$

$$\langle T[G_{\mu\nu}(x) G^+_{\lambda\rho}(y)] \rangle_{vac.}$$
$$= -m^{-2}[\delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\rho}\partial_\mu\partial_\lambda - \delta_{\mu\rho}\partial_\nu\partial_\lambda$$
$$- \delta_{\nu\lambda}\partial_\mu\partial_\rho]\Delta_F(x-y) \quad . \qquad (3.12)$$

The resulting Feynman diagrams obtained by using the standard procedure of Dyson and Wick[13] are listed in Table 1.

4. ξ -Limiting Formalism

Although we have now a covariant theory, it is still not renormalisable. To remove this difficulty we use the ξ - limiting technique in our formulation of the vector meson field theory.

4.1. Lagrangian

In this formalism we add two more terms to the Lagrangian (3.1) proportional to a dimensionless parameter ξ and then take the limit $\xi \to 0$.

$$\mathcal{L} = \partial_\mu G^+_{\mu\nu}\,\partial_\lambda G_{\lambda\nu} + \tfrac{1}{2} m^2 G^+_{\mu\nu} G_{\mu\nu}$$
$$+ \xi\left(\tfrac{1}{2}\partial_\lambda G^+_{\mu\nu}\,\partial_\lambda G_{\mu\nu} - \partial_\lambda G^+_{\mu\nu}\,\partial_\mu G_{\lambda\nu}\right) \tag{4.1}$$

The equation of motion that follows from the Lagrangian (4.1) is

$$\partial_\mu\partial_\lambda G_{\lambda\nu} - \partial_\nu\partial_\lambda G_{\lambda\mu} - m^2 G_{\mu\nu} + \xi\,\epsilon_{\mu\nu\lambda\rho}\,\partial_\lambda U_\rho = 0 \tag{4.2}$$

where

$$U_\rho = \partial_\lambda G^D_{\lambda\rho} \quad , \quad G^D_{\lambda\rho} = \tfrac{1}{2}\epsilon_{\lambda\rho\mu\nu} G_{\mu\nu} .$$

The free Hamiltonian H_0 is given by

$$H_0 = -\underset{\sim}{\Pi}_4^+\cdot\underset{\sim}{\Pi}_4 + i(\underset{\sim}{\nabla}\times\underset{\sim}{\Pi}_4)\cdot\underset{\sim}{G} + i(\underset{\sim}{\nabla}\times\underset{\sim}{\Pi}_4^+)\cdot\underset{\sim}{G}^+ - (\underset{\sim}{\nabla}\cdot\underset{\sim}{G}_4^+)\cdot(\underset{\sim}{\nabla}\cdot\underset{\sim}{G}_4)$$
$$- m^2\underset{\sim}{G}_4^+\cdot\underset{\sim}{G}_4 - m^2\underset{\sim}{G}^+\cdot\underset{\sim}{G} - \xi^{-1}\underset{\sim}{\Pi}^+\cdot\underset{\sim}{\Pi} + i\,\underset{\sim}{\Pi}\cdot(\underset{\sim}{\nabla}\times\underset{\sim}{G}_4)$$
$$+ i\,\underset{\sim}{\Pi}^+\cdot(\underset{\sim}{\nabla}\times\underset{\sim}{G}_4^+) - \xi(\underset{\sim}{\nabla}\cdot\underset{\sim}{G}^+)(\underset{\sim}{\nabla}\cdot\underset{\sim}{G}) . \tag{4.3}$$

In the expression (4.3)

$\underset{\sim}{\Pi}_4 \equiv \Pi_{4k}$, $\underset{\sim}{G}_4 \equiv G_{4k}$, $k = 1, 2, 3$;

$G_{ik} \equiv G_l$, $\Pi_{ik} \equiv \Pi_l$, i, k, l are cyclic ,

$\Pi^+_{4k} = -\Pi^*_{4k}$, $G^+_{4k} = -G^*_{4k}$,

$\Pi_k^+ = \Pi_k^*$, $G_k^+ = G_k^*$;

$\Pi_{\mu\nu}$ and $\Pi^+_{\mu\nu}$ are, respectively, the momenta conjugate to

$G_{\mu\nu}$ and $G^{+}_{\mu\nu}$ and are given by

$$\Pi_{\mu\nu} = -i\delta_{4\mu}\partial_\lambda G^{+}_{\lambda\nu} + i\delta_{4\nu}\partial_\lambda G^{+}_{\lambda\mu} - i\xi\partial_4 G^{+}_{\mu\nu}$$
$$+ i\xi\partial_\mu G^{+}_{4\nu} - i\xi\partial_\nu G^{+}_{4\mu} \quad , \tag{4.4}$$

$$\Pi^{+}_{\mu\nu} = -i\delta_{4\mu}\partial_\lambda G_{\lambda\nu} + i\delta_{4\nu}\partial_\lambda G_{\lambda\mu} - i\xi\partial_4 G^{+}_{\mu\nu}$$
$$+ i\xi\partial_\mu G_{4\nu} - i\xi\partial_\nu G_{4\mu} \quad , \tag{4.5}$$

The free fields correspond to a system of uncoupled vector mesons and pseudovector mesons and in momentum space they have the following representation:

$$\underset{\sim}{G}_4 = \sum_{\underset{\sim}{k},r}(2\Omega\omega)^{-\frac{1}{2}}\left[a^{(r)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) + b^{*(r)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t)\right](\omega/m)\,\underset{\sim}{e}^{(r)}_{\underset{\sim}{k}}$$

$$+ \sum_{\underset{\sim}{k}}(2\Omega\omega)^{-\frac{1}{2}}\left[a^{(3)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) + b^{*(3)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t)\right]\hat{\underset{\sim}{k}}$$

$$+ \sum_{\underset{\sim}{k}}(2\Omega\nu)^{-\frac{1}{2}}\left[c^{(1)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) + d^{*(1)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t)\right]\left(-|\underset{\sim}{k}|/m\right)\underset{\sim}{e}^{(2)}_{\underset{\sim}{k}}$$

$$+ \sum_{\underset{\sim}{k}}(2\Omega\nu)^{-\frac{1}{2}}\left[c^{(2)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) + d^{*(2)}_{\underset{\sim}{k}}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t)\right]\left(|\underset{\sim}{k}|/m\right)\underset{\sim}{e}^{(1)}_{\underset{\sim}{k}}$$

$$\underset{\sim}{\Pi}_4 = \sum_{\underset{\sim}{k},r} i(2\Omega\omega)^{-\frac{1}{2}}\left[a^{*(r)}_{\underset{\sim}{k}}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) - b^{(r)}_{\underset{\sim}{k}}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t)\right] m\,\underset{\sim}{e}^{(r)}_{\underset{\sim}{k}}$$

$$+ \sum_{\underset{\sim}{k}} i(2\Omega\omega)^{-\frac{1}{2}}\left[a^{*(3)}_{\underset{\sim}{k}}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) - b^{(3)}_{\underset{\sim}{k}}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t)\right]\omega\,\hat{\underset{\sim}{k}}$$

$$\underset{\sim}{\Pi} = \sum_{\underset{\sim}{k}, r} (2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{*(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) + d_{\underset{\sim}{k}}^{(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) \right] m\, \underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+ \sum_{\underset{\sim}{k}} (2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{*(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) + d_{\underset{\sim}{k}}^{(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) \right] (m\nu/m')\, \hat{\underset{\sim}{k}}$$

$$\underset{\sim}{G} = \sum_{\underset{\sim}{k}} i\,(2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{(1)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) - b_{\underset{\sim}{k}}^{*(1)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) \right] \left(-|\underset{\sim}{k}|/m\right) \underset{\sim}{e}_{\underset{\sim}{k}}^{(2)}$$

$$+ \sum_{\underset{\sim}{k}} i\,(2\Omega\omega)^{-\frac{1}{2}} \left[a_{\underset{\sim}{k}}^{(2)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t) - b_{\underset{\sim}{k}}^{*(2)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\omega t) \right] \left(|\underset{\sim}{k}|/m\right) \underset{\sim}{e}_{\underset{\sim}{k}}^{(1)}$$

$$+ \sum_{\underset{\sim}{k}, r} i\,(2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{(r)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) - d_{\underset{\sim}{k}}^{*(r)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) \right] (\nu/m)\, \underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+ \sum_{\underset{\sim}{k}} i\,(2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{(3)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) - d_{\underset{\sim}{k}}^{*(3)} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) \right] (m'/m)\, \hat{\underset{\sim}{k}} \tag{4.6}$$

where

$$\omega = (\underset{\sim}{k}^2 + m^2)^{\frac{1}{2}} > 0 \;, \quad \nu = (\underset{\sim}{k}^2 + m'^2)^{\frac{1}{2}} > 0 \;,$$

$$m'^2 = \xi^{-1} m^2 \quad \text{and} \quad \hat{\underset{\sim}{k}} = \underset{\sim}{k}\, |\underset{\sim}{k}|^{-1}$$

In the expansion (4.6) $c_{\underset{\sim}{k}}^{(r)}$ $(r = 1,2)$ and $c_{\underset{\sim}{k}}^{(3)}$ are, respectively, the annihilation operators for transverse and longitudinal pseudovector mesons. $c_k^{*(r)}$ $(r = 1,2)$ and $c_k^{*(3)}$ are the corresponding creation operators. $d_{\underset{\sim}{k}}$ and $d_{\underset{\sim}{k}}^{*}$ are, respectively, the annihilation and creation operators for the negatively charged pseudovector mesons.

By substituting the formulae (4.6) into the expression for the free Hamiltonian (4.3) one obtains

$$H_0 = \sum_{\underset{\sim}{k},r} \omega \left(a_{\underset{\sim}{k}}^{*(r)} a_{\underset{\sim}{k}}^{(r)} + \tfrac{1}{2} \right) + \sum_{\underset{\sim}{k}} \omega \left(a_{\underset{\sim}{k}}^{*(3)} a_{\underset{\sim}{k}}^{(3)} + \tfrac{1}{2} \right)$$
$$- \sum_{\underset{\sim}{k},r} \nu \left(c_{\underset{\sim}{k}}^{*(r)} c_{\underset{\sim}{k}}^{(r)} + \tfrac{1}{2} \right) - \sum_{\underset{\sim}{k}} \nu \left(c_{\underset{\sim}{k}}^{*(3)} c_{\underset{\sim}{k}}^{(3)} + \tfrac{1}{2} \right)$$

+ same terms with $a \rightarrow b$ and $c \rightarrow d$ (4.7)

The equal time commutation relations are given by

$$\left[\Pi_{\mu\nu} (\underset{\sim}{r}, t), G_{\lambda\rho} (\underset{\sim}{r}', t) \right] = -i \left(\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda} \right) \delta^3 (\underset{\sim}{r} - \underset{\sim}{r}'),$$
$$\left[\Pi_{\mu\nu}^{+} (\underset{\sim}{r}, t), G_{\lambda\rho}^{+} (\underset{\sim}{r}', t) \right] = -i \left(\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda} \right) \delta^3 (\underset{\sim}{r} - \underset{\sim}{r}').$$

(4.8)

The free field propagator consists of two parts, the first part corresponding to the vector and the second part to the pseudo-vector meson, and is explicitly given by

$$\begin{aligned} S_{\mu\nu,\lambda\rho} = \; & i(p^2 + m^2 - i\epsilon)^{-1} \left[\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda} p_\nu p_\rho / m^2 \right. \\ & - \delta_{\nu\rho} p_\mu p_\lambda / m^2 + \delta_{\mu\rho} p_\nu p_\lambda / m^2 + \delta_{\nu\lambda} p_\mu p_\rho / m^2 - \delta_{\mu\rho}\delta_{\nu\lambda} p^2/m^2 \\ & \left. + \delta_{\mu\lambda}\delta_{\nu\rho} p^2/m^2 \right] - i \left(p^2 + \xi^{-1} m^2 + i\epsilon \right)^{-1} \left[- \delta_{\mu\lambda} p_\nu p_\rho / m^2 \right. \\ & - \delta_{\nu\rho} p_\mu p_\lambda / m^2 + \delta_{\mu\rho} p_\nu p_\lambda / m^2 + \delta_{\nu\lambda} p_\mu p_\rho / m^2 - \delta_{\mu\rho}\delta_{\nu\lambda} p^2/m^2 \\ & \left. + \delta_{\mu\lambda}\delta_{\nu\rho} p^2/m^2 \right]. \end{aligned}$$

(4.9)

From the above expression for the propagator in momentum space it may appear that the pseudovector part of it will act like a regulator to the divergent terms in the first part. But in fact it is not so, because the denominator of the first part contains $(p^2+m^2-i\epsilon)$ and that of the second part has $(p^2+m^2\xi^{-1}+i\epsilon)$. Apart from this, the Hamiltonian is now indefinite - as the pseudovector mesons appear in our theory with negative energy.

4.2. Negative Metric

In order to make the free Hamiltonian positive definite and also to change the sign of $(i\epsilon)$ in the pseudovector part of the propagator, a negative metric is introduced in the Hilbert space. We use the identical Lagrangian given by expression (4.1) except that the hermitian conjugate operators are now replaced by

$$G_{\mu\nu}^{\blacklozenge} \equiv \eta^{-1} G_{\mu\nu}^{+} \eta ,$$
$$\Pi_{\mu\nu}^{\blacklozenge} \equiv \eta^{-1} \Pi_{\mu\nu}^{+} \eta \tag{4.10}$$

where η is the metric of the Hilbert space and is given by

$$\eta = \exp i\pi \Big[\sum_{\underset{\sim}{k},r} \left(c_{\underset{\sim}{k}}^{*(r)} c_{\underset{\sim}{k}}^{(r)} + d_{\underset{\sim}{k}}^{*(r)} d_{\underset{\sim}{k}}^{(r)} \right) + \sum_{\underset{\sim}{k}} \left(c_{\underset{\sim}{k}}^{*(3)} c_{\underset{\sim}{k}}^{(3)} + d_{\underset{\sim}{k}}^{*(3)} d_{\underset{\sim}{k}}^{(3)} \right) \Big] . \tag{4.11}$$

The free field Hamiltonian and the equal time commutation relations are given by the expressions (4.3) and (4.8), respectively, with $G_{\mu\nu}^{+}$ and $\Pi_{\mu\nu}^{+}$ replaced by $G_{\mu\nu}^{\blacklozenge}$ and $\Pi_{\mu\nu}^{\blacklozenge}$, respectively.

After the change of metric the field operators have the following representation in momentum space:

$$\underset{\sim}{G}_4 = \sum_{\underset{\sim}{k},r} (2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{(r)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t) + b_{\underset{\sim}{k}}^{*(r)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)\right](\omega/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+\sum_{\underset{\sim}{k}}(2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{(3)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)+b_{\underset{\sim}{k}}^{*(3)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)\right]\hat{\underset{\sim}{k}}$$

$$+\sum_{\underset{\sim}{k}}(2\Omega\nu)^{-\frac{1}{2}}\left[c_{\underset{\sim}{k}}^{(1)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\nu t)+d_{\underset{\sim}{k}}^{*(1)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\nu t)\right](|\underset{\sim}{k}|/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(2)}$$

$$+\sum_{\underset{\sim}{k}}(2\Omega\nu)^{-\frac{1}{2}}\left[c_{\underset{\sim}{k}}^{(2)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\nu t)+d_{\underset{\sim}{k}}^{*(2)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\nu t)\right](-|\underset{\sim}{k}|/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(1)}$$

$$\underset{\sim}{\Pi}_4 = \sum_{\underset{\sim}{k},r} i(2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{*(r)}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)-b_{\underset{\sim}{k}}^{(r)}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)\right]m\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+\sum_{\underset{\sim}{k}} i(2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{*(3)}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)-b_{\underset{\sim}{k}}^{(3)}\exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)\right]\omega\,\hat{\underset{\sim}{k}}$$

$$\underset{\sim}{G} = \sum_{\underset{\sim}{k}} i(2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{(1)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)-b_{\underset{\sim}{k}}^{*(1)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)\right](-|\underset{\sim}{k}|/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(2)}$$

$$+\sum_{\underset{\sim}{k}} i(2\Omega\omega)^{-\frac{1}{2}}\left[a_{\underset{\sim}{k}}^{(2)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)-b_{\underset{\sim}{k}}^{*(2)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t)\right](|\underset{\sim}{k}|/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(1)}$$

$$+\sum_{\underset{\sim}{k},r} i(2\Omega\nu)^{-\frac{1}{2}}\left[c_{\underset{\sim}{k}}^{(r)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\nu t)-d_{\underset{\sim}{k}}^{*(r)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\nu t)\right](\nu/m)\,\underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$

$$+\sum_{\underset{\sim}{k}} i(2\Omega\nu)^{-\frac{1}{2}}\left[c_{\underset{\sim}{k}}^{(3)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\nu t)-d_{\underset{\sim}{k}}^{*(3)}\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\nu t)\right](m'/m)\,\hat{\underset{\sim}{k}}$$

$$\underset{\sim}{\Pi} = \sum_{\underset{\sim}{k},r} (2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{*(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) + d_{\underset{\sim}{k}}^{(r)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) \right] m\, \underset{\sim}{e}_{\underset{\sim}{k}}^{(r)}$$
$$+ \sum_{\underset{\sim}{k}} (2\Omega\nu)^{-\frac{1}{2}} \left[c_{\underset{\sim}{k}}^{*(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} + i\nu t) + d_{\underset{\sim}{k}}^{(3)} \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\nu t) \right] (m\nu/m')\, \hat{\underset{\sim}{k}}$$

(4.12)

The free Hamiltonian, which is now positive definite, is given by

$$H_0 = \sum_{\underset{\sim}{k},r} \left(a_{\underset{\sim}{k}}^{*(r)} a_{\underset{\sim}{k}}^{(r)} + \tfrac{1}{2}\right) + \sum_{\underset{\sim}{k}} \left(a_{\underset{\sim}{k}}^{*(3)} a_{\underset{\sim}{k}}^{(3)} + \tfrac{1}{2}\right)$$
$$+ \sum_{\underset{\sim}{k},r} \left(c_{\underset{\sim}{k}}^{*(r)} c_{\underset{\sim}{k}}^{(r)} + \tfrac{1}{2}\right) + \sum_{\underset{\sim}{k}} \left(c_{\underset{\sim}{k}}^{*(3)} c_{\underset{\sim}{k}}^{(3)} + \tfrac{1}{2}\right)$$

(4.13)

+ same terms with $a \rightarrow b$ and $c \rightarrow d$.

The change of metric also changes the sign of $(i\epsilon)$ in the pseudo-vector part of the propagator (4.9). So the ξ-dependent terms in the propagator act like regulators.

5.1. Interactions in the ξ-limiting Formalism

We shall discuss a vector meson field with arbitrary magnetic moment and quadrupole moment in interaction with the electromagnetic field.

The Lagrangian and the equations of motion are given by

$$\mathcal{L} = \partial_\mu^+ G_{\mu\nu}^+ \partial_\lambda^- G_{\lambda\nu} + \tfrac{1}{2} m^2 G_{\mu\nu}^+ G_{\mu\nu} + \xi\left(\tfrac{1}{2}\partial_\lambda^+ G_{\mu\nu}^+ \partial_\lambda^- G_{\mu\nu} - \partial_\lambda^+ G_{\mu\nu}^+ \partial_\mu^- G_{\lambda\nu}\right) + iek F_{\mu\nu} G_{\mu\rho}^+ G_{\rho\nu} \; . \qquad (5.1)$$

$$\partial_{\bar{\mu}}(\partial_{\bar{\lambda}} G_{\lambda\nu}) - \partial_{\bar{\nu}}(\partial_{\bar{\lambda}} G_{\lambda\mu}) - m^2 G_{\mu\nu} + \xi \epsilon_{\mu\nu\lambda\rho}\partial_\lambda U_\rho$$
$$+ iek(F_{\mu\lambda}G_{\lambda\nu} - F_{\nu\lambda}G_{\lambda\mu}) = 0 \tag{5.2}$$

Now by a unitary transformation we go over to the interaction picture where the space-time dependence of the field operators is the same as in the free case. The metric η of the Hilbert space is still given by expression (4.11) in the interaction picture.

The Hamiltonian in the interaction picture is

$$H = H_0 + H_{int.}$$

where

$$\begin{aligned} H_{int.} = {} & ie\varphi_\lambda(\partial_\mu G_{\mu\nu}^\blacklozenge G_{\lambda\nu} - \partial_\mu G_{\mu\nu} G_{\lambda\nu}^\blacklozenge) \\ & - e^2\varphi_\mu\varphi_\nu G_{4\mu}^\blacklozenge G_{4\nu} + ie\xi\varphi_\lambda G_{\mu\nu}(\tfrac{1}{2}\partial_\lambda G_{\mu\nu}^\blacklozenge - \partial_\mu G_{\lambda\nu}^\blacklozenge) \\ & - ie\xi\varphi_\lambda G_{\mu\nu}^\blacklozenge(\tfrac{1}{2}\partial_\lambda G_{\mu\nu} - \partial_\mu G_{\lambda\nu}) + e^2\xi\varphi_i\varphi_l G_{ik}^\blacklozenge G_{lk} \\ & - \tfrac{1}{2}e^2\xi\varphi_l\varphi_l G_{ik}^\blacklozenge G_{ik} - iekF_{\mu\nu}G_{\mu\rho}^\blacklozenge G_{\rho\nu} \\ & + \tfrac{1}{2}e^2k^2(G_{4\rho}^\blacklozenge G_{\rho l} - G_{l\rho}^\blacklozenge G_{\rho 4})^2 \end{aligned} \tag{5.3}$$

H_{int} is not hermitian and consequently the S-matrix is not unitary. But they satisfy the following relations

$$\eta^{-1} H_{int}^{+} \eta = H_{int}\,, \tag{5.4}$$
$$\eta^{-1} S^{+} \eta = S^{-1}\,. \tag{5.5}$$

Now if we assume that the total energy of the system is always less than $\xi^{-\frac{1}{2}} m$, then there can be no pseudovector mesons either in the initial or final state. So $\eta = 1$ and the S-matrix is truly unitary provided that $E < \xi^{-\frac{1}{2}} m$.

5.2. Feynman Diagrams and Renormalisability

Feynman diagrams for such a theory can be written down by using the Dyson-Wick procedure. The expressions for the propagators and vertices are listed in Table II.

At high energies the propagator for the vector meson varies asymptotically like P^{-2} for $\xi > 0$. So all the divergences in the higher order diagrams can be renormalised in the same way as in the theory of charged scalar mesons interacting with the electromagnetic field.

Appendix I

Calculation of Magnetic and Quadrupole Moments.

To the approximation in which the magnetic field $\underset{\sim}{H}$ is constant in space and time and the gauge of the electromagnetic field is chosen in such a way that $\varphi_4 = 0$ and also the second and higher powers of the magnetic field are neglected, the magnetic moment can be defined by the space vector $\underset{\sim}{M}$ given by

$$\underset{\sim}{H} \cdot \underset{\sim}{M} = \int d^3x \, L_1 \tag{A1}$$

where L_1 is the interaction Lagrangian.

The contribution from the first term in expression (1.7) to the magnetic moment of the vector meson is $-e\kappa/2m$ [12] and that from the second term is given by

$$\underset{\sim}{H} \cdot \underset{\sim}{M} = -ie\kappa \int d^3x \left(H_{in} G_{il}^{+} G_{nl} + H_{in} G_{i4}^{+} G_{n4} \right) \tag{A2}$$

where $H_{in} \equiv H_l$; i, n, l are cyclic.

Expanding the vector meson field in terms of the annihilation and creation operators and considering the limiting case where the mesons are at rest, one obtains from the expression (A2)

$$M_3 = (-ie\kappa/2m)\left(-a_0^{*(1)} a_0^{(2)} + a_0^{*(2)} a_0^{(1)} + b_0^{*(1)} b_0^{(2)} - b_0^{*(2)} b_0^{(1)} \right) \tag{A3}$$

where M_3 is the magnetic moment along the z -axis.

Expression (A3) can further be diagonalised(14) and the magnetic moment of the vector meson is given by

$$\underset{\sim}{M} = -ek/2m\, \underset{\sim}{S} \qquad \text{for positive mesons}$$

$$\underset{\sim}{M} = ek/2m\, \underset{\sim}{S} \qquad \text{for negative mesons} \tag{A4}$$

The quadrupole moment Q is defined by

$$Q = \int (3z^2 - r^2)\rho\, d^3x \tag{A5}$$

where ρ is the static charge density for the state $S_z = +1$.

The current density J_μ due to the interaction Lagrangian $iek F_{\mu\nu} G^{+}_{\mu\rho} G_{\rho\nu}$ is

$$J_\mu = iek\, \partial_\nu \left(G^{+}_{\mu\rho} G_{\rho\nu} - G^{+}_{\nu\rho} G_{\rho\mu} \right) \tag{A6}$$

By substituting the value of ρ in expression (A5) and taking into account that ρ is the charge density in the state $S_z = 1$ one obtains

$$Q = ek/m^2 \tag{A7}$$

APPENDIX II

The Stueckelberg Formalism in the Inverted Scheme

In the second order formulation the following equation of motion

$$(\Box - m^2) G_{\mu\nu} = 0 \tag{A1}$$

together with the subsidiary condition

$$\partial_\lambda G_{\mu\nu} + \partial_\mu G_{\nu\lambda} + \partial_\nu G_{\lambda\mu} = 0 \tag{A2}$$

where $G_{\mu\nu}$ is an antisymmetric tensor, describes the spin one particles. Of the four equations contained in (5.2) only three are essentially independent so that the tensor $G_{\mu\nu}$ has only three independent components corresponding to the three states of polarisation of the spin one particles.

The equation of motion (5.1) and the subsidiary condition (5.2) may be derived from the following Lagrangian:

$$\mathcal{L} = \partial_\mu G_{\mu\nu}^{+} \partial_\lambda G_{\lambda\nu} + \tfrac{1}{2} m^2 G_{\mu\nu}^{+} G_{\mu\nu} . \tag{A3}$$

The dynamic equation is

$$\partial_\mu (\partial_\lambda G_{\lambda\nu}) - \partial_\nu (\partial_\lambda G_{\lambda\mu}) - m^2 G_{\mu\nu} = 0 \tag{A4}$$

and by multiplying eqn. (A4) with $\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_\rho$ we get the subsidiary condition (A2).

In the Stueckelberg formalism the field $G_{\mu\nu}$ is decomposed into two parts:

$$G_{\mu\nu} = B_{\mu\nu} + \frac{1}{m} \epsilon_{\mu\nu\lambda\rho} \partial_\lambda B_\rho \, ,$$

$$G^{+}_{\mu\nu} = B^{+}_{\mu\nu} + \frac{1}{m} \epsilon_{\mu\nu\lambda\rho} \partial_\lambda B^{+}_\rho \, ,$$

$$G^{+}_{ik} = G^{*}_{ik} \, , \quad G^{+}_{i4} = - G^{*}_{i4} \, ,$$

$$B_i^{+} = - B_i^{*} \, , \quad B_4^{+} = B_4^{*} \, . \tag{A5}$$

The Lagrangian in this theory is

$$\mathcal{L} = -\frac{1}{2} \partial_\lambda B^{+}_{\mu\nu} \partial_\lambda B_{\mu\nu} - \frac{1}{2} m^2 B^{+}_{\mu\nu} B_{\mu\nu}$$
$$- \frac{1}{2} \left(\partial_\mu B_\nu^{+} - \partial_\nu B_\mu^{+} \right) \left(\partial_\mu B_\nu - \partial_\nu B_\mu \right) - m^2 B^{+}_\mu B_\mu \, . \tag{A6}$$

which gives the wave equations

$$(\Box - m^2) B_{\mu\nu} = 0 \quad , \tag{A7}$$

$$\partial_\mu \left(\partial_\mu B_\nu - \partial_\nu B_\mu \right) - m^2 B_\nu = 0 \, . \tag{A8}$$

From equation (5.8) we can also derive

$$(\Box - m^2) B_\mu = 0 \quad , \tag{A9}$$

$$\partial_\mu B_\mu = 0 \, . \tag{A10}$$

If we now assume the following initial conditions on a flat space-like surface at time t_1 :

$$\left(\partial_\nu B^D_{\nu\mu} + m B_\mu\right)|\Psi\rangle = 0 \quad \text{at } t = t_1 \quad , \tag{A11}$$

$$\partial_4\left(\partial_\nu B^D_{\nu\mu} + m B_\mu\right)|\Psi\rangle = 0 \quad \text{at } t = t_1 \, , \tag{A12}$$

where $B^D_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} B_{\lambda\rho}$,

then by virtue of the wave equations (A7) and (A9) we get the subsidiary condition

$$\left(\partial_\nu B^D_{\nu\mu} + m B_\mu\right)|\Psi\rangle = 0 \qquad \text{at all times} \tag{A13}$$

The free field commutation relations are

$$\left[B^+_{\mu\nu}(x), B_{\lambda\rho}(y)\right] = \left(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}\right)\Delta(x-y) \, ,$$

$$\left[U^+_\mu(x), U_\nu(y)\right] = \left(\delta_{\mu\nu} - \partial_\mu\partial_\nu/m^2\right)\Delta(x-y) \, ,$$

$$\left[B^+_{\mu\nu}(x), B_\lambda(y)\right] = \left[B_{\mu\nu}(x), B^+_\lambda(y)\right] = 0 \, . \tag{A14}$$

The commutation relations (A14) are consistent with the equations of motion (A7) and (A9) as well as with the subsidiary condition (A13).

References

1) T.D. Lee & C.N. Yang, Phys. Rev. 128 (1962) 885.

2) N. Kemmer, Helv. Phys. Acta 33 (1960) 892.

3) N.N. Bogoliubov & D.V. Shirkov, Introduction to the Theory of Quantised Fields (Interscience, New York, 1959), p. 340.

4) H. Umezawa, Quantum Field Theory (North-Holland, Amsterdam), p. 207.

5) S.L. Glashow, Nuc. Phys. 10 (1959) 107.

6) P.P. Divakaran, Nuc. Phys. 42 (1963) 235.

7) S. Kamefuchi, O'Raifeartaigh, A. Salam, Nuc. Phys. 28 (1961), 529.

8) J.S.R. Chisholm, Nuc. Phys. 26 (1961) 469.

9) S. Kamefuchi, Nuc. Phys. 18 (1960) 691.

10) A. Salam, Nuc. Phys. 18 (1960) 681.

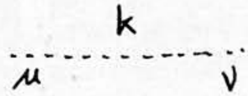
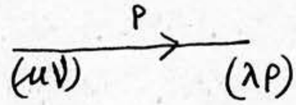
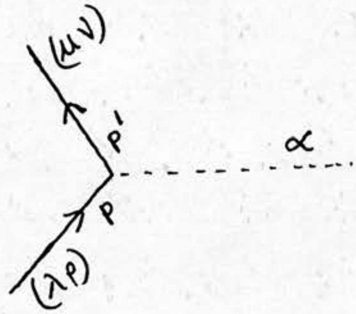
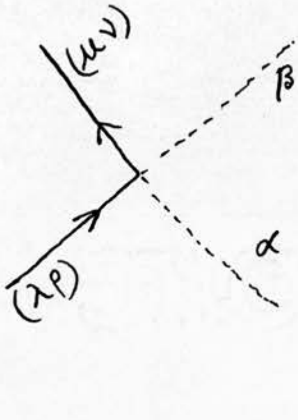
11) N. Kemmer, Proc. Roy. Soc. A173 (1939) 91.

12) W. Pauli, Rev. Mod. Phys. 13 (1941) 203.

13) F.J. Dyson, Phys. Rev. 75 (1949) 486.
G.C. Wick, Phys. Rev. 80 (1950) 268.

14) G. Wentzel, Quantum Theory of Fields (Interscience, New York, 1949), p. 88.

TABLE 1. Feynman Diagram in Momentum Representation for Lagrangian (2.1) with ∂_μ replaced by $\partial_\mu - ie\varphi_\mu$.

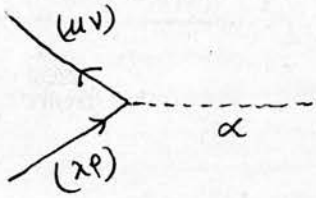
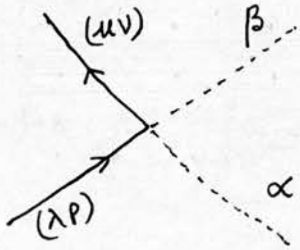
Element	Graph	Value
Internal Photon Line		$D = -i\,\delta_{\mu\nu}\,(k^2)^{-1}$
Internal Meson Line		$S = i(p^2+m^2-i\epsilon)^{-1}m^2\left[-\delta_{\mu\lambda}p_\nu p_\rho - \delta_{\nu\rho}p_\mu p_\lambda + \delta_{\mu\rho}p_\nu p_\lambda + \delta_{\nu\lambda}p_\mu p_\rho\right]$
3-Vertex		$V = -ie\left[p'_\mu(\delta_{\alpha\lambda}\delta_{\nu\rho} - \delta_{\alpha\rho}\delta_{\nu\lambda}) - p'_\nu(\delta_{\alpha\lambda}\delta_{\mu\rho} - \delta_{\alpha\rho}\delta_{\mu\lambda}) + p_\lambda(\delta_{\alpha\mu}\delta_{\nu\rho} - \delta_{\alpha\nu}\delta_{\mu\rho}) - p_\rho(\delta_{\alpha\mu}\delta_{\nu\lambda} - \delta_{\alpha\nu}\delta_{\mu\lambda})\right]$
4-Vertex		$U = ie^2(\delta_{\mu\alpha}\delta_{\lambda\beta}\delta_{\nu\rho} + \delta_{\mu\beta}\delta_{\alpha\lambda}\delta_{\nu\rho} - \delta_{\nu\alpha}\delta_{\lambda\beta}\delta_{\mu\rho} - \delta_{\nu\beta}\delta_{\alpha\lambda}\delta_{\mu\rho} - \delta_{\mu\alpha}\delta_{\rho\beta}\delta_{\nu\lambda} - \delta_{\mu\beta}\delta_{\rho\alpha}\delta_{\nu\lambda} + \delta_{\nu\beta}\delta_{\rho\alpha}\delta_{\mu\lambda} + \delta_{\alpha\nu}\delta_{\rho\beta}\delta_{\mu\lambda})$

The weight of each diagram is S^{-1} where S is the number of diagrams with identical topological structure.

TABLE 2. Feynman Diagram in Momentum Space for Lagrangian (4.1) with a Negative Metric.

Element	Graph	Value
Internal Photon Line	$\mu \overset{k}{\text{- - - - - - - - -}} \nu$	$D = -i\delta_{\mu\nu}(k^2)^{-1}$
Internal Meson Line	$(\mu\nu) \xrightarrow{\quad p \quad} (\lambda\rho)$	$S = i(p^2+m^2-i\epsilon)^{-1}\big[\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda}p_\nu p_\rho/m^2 - \delta_{\nu\rho}p_\mu p_\lambda/m^2 + \delta_{\mu\rho}p_\nu p_\lambda/m^2 + \delta_{\nu\lambda}p_\mu p_\rho/m^2 - \delta_{\mu\rho}\delta_{\nu\lambda}p^2/m^2 + \delta_{\mu\lambda}\delta_{\nu\rho}p^2/m^2\big]$ $- i(p^2+\xi^{-1}m^2-i\epsilon)^{-1}m^{-2}\big[-\delta_{\mu\lambda}p_\nu p_\rho - \delta_{\nu\rho}p_\mu p_\lambda + \delta_{\mu\rho}p_\nu p_\lambda + \delta_{\nu\lambda}p_\mu p_\rho - \delta_{\mu\rho}\delta_{\nu\lambda}p^2 + \delta_{\mu\lambda}\delta_{\nu\rho}p^2\big]$.

TABLE 2 (Contd.)

Element	Graph	Value
3-Vertex		$V = ie\Big[(\delta_{\alpha\lambda}\delta_{\nu\rho} - \delta_{\alpha\rho}\delta_{\nu\lambda})\big\{(\kappa-\xi)(p'-p)_\mu - (1-\xi)p'_\mu\big\} - (\delta_{\alpha\lambda}\delta_{\mu\rho} - \delta_{\alpha\rho}\delta_{\mu\lambda})\cdot\big\{(\kappa-\xi)(p'-p)_\nu - (1-\xi)p'_\nu\big\} - (\delta_{\alpha\mu}\delta_{\nu\rho} - \delta_{\nu\alpha}\delta_{\mu\rho})\big\{(\kappa-\xi)(p'-p)_\lambda + (1-\xi)p_\lambda\big\} + (\delta_{\alpha\mu}\delta_{\nu\lambda} - \delta_{\alpha\nu}\delta_{\mu\lambda})\big\{(\kappa-\xi)(p'-p)_\rho + (1-\xi)p_\rho\big\}\Big] - ie\xi(\delta_{\lambda\mu}\delta_{\rho\nu} - \delta_{\lambda\nu}\delta_{\rho\mu})(p'+p)_\alpha$
4-Vertex		$U = ie^2(1-\xi)\big[\delta_{\mu\alpha}\delta_{\lambda\beta}\delta_{\nu\rho} + \delta_{\mu\beta}\delta_{\lambda\alpha}\delta_{\nu\rho} - \delta_{\nu\alpha}\delta_{\lambda\beta}\delta_{\mu\rho} - \delta_{\nu\beta}\delta_{\lambda\alpha}\delta_{\mu\rho} - \delta_{\mu\alpha}\delta_{\rho\beta}\delta_{\nu\lambda} - \delta_{\mu\beta}\delta_{\rho\alpha}\delta_{\nu\lambda} + \delta_{\nu\alpha}\delta_{\rho\beta}\delta_{\mu\lambda} + \delta_{\nu\beta}\delta_{\rho\alpha}\delta_{\mu\lambda}\big] + 2ie^2\xi\big[\delta_{\lambda\mu}\delta_{\rho\nu} - \delta_{\rho\mu}\delta_{\lambda\nu}\big]\delta_{\alpha\beta}$

All the diagrams are weighted as in Table 1.

PART II

THE MINIMAL ELECTROMAGNETIC INTERACTIONS OF THE CHARGED SPIN TWO FIELD

PART II

THE MINIMAL ELECTROMAGNETIC INTERACTIONS OF THE CHARGED SPIN 2 FIELD

Introduction

The theory of higher spin fields was proposed by Dirac[1), Fierz[2), and Fierz and Pauli[3). It was further developed and simplified by the works of Rarita and Schwinger[4), Gupta[5), Moldaur and Case[6), and Fronsdal[7). In general, particles with integral spin $s \geqslant 1$ are described by a symmetric tensor field which satisfies the Klein-Gordan equation and a set of subsidiary conditions. More explicitly, particles with integral spin s are described by the following set of equations:

$$(\Box - m^2)\psi_{\mu_1 \cdots \mu_s} = 0, \tag{1.1}$$

$$\partial_\mu \psi_{\mu\mu_2 \cdots \mu_s} = 0, \tag{1.2}$$

$$\psi_{\mu\mu\mu_3 \cdots \mu_s} = 0, \tag{1.3}$$

where $\psi_{\mu_1 \cdots \mu_s}$ is a symmetric tensor of rank s.

Dirac[1) was the first to discuss the interaction of these higher spin fields with the Maxwell field. He introduced the electromagnetic interaction by the gauge invariant replacement, $\partial_\mu \to \pi_\mu = \partial_\mu - ie\varphi_\mu$, where φ_μ is the electromagnetic potential, in the equation of motion as well as in the subsidiary conditions. The operators π_μ and π^2 do not, in general, commute. So equations (1.1) and (1.2) become inconsistent with each other after the introduction of electromagnetic interactions. Then Fierz and Pauli[3)] proposed a method of introducing the electromagnetic interaction with the higher spin fields, which removes the inconsistency between the field equations and the subsidiary conditions in the presence of electromagnetic interactions. This method consists essentially in finding a suitable Lagrangian from which all the subsidiary conditions and the field equations can be derived with the help of a variational principle; and then introducing the electromagnetic interaction in this Lagrangian. Fierz and Pauli[3)] discussed explicitly the theory of the spin 2 field interacting with the Maxwell field, where they introduced in the interaction Lagrangian a term containing the direct interaction of the electromagnetic field with the spin 2 field, besides those containing the minimal electromagnetic interaction. Later, Federbush[8),9)], pointed out that the minimal electromagnetic interaction of the spin 2 field is inconsistent, a term in the interaction Lagrangian containing the direct interaction of the spin 2 field with the electromagnetic field being essential.

We have shown here that it is not necessary to introduce in the interaction Lagrangian the direct interaction of the spin 2 field with the electromagnetic field. One can deduce, in the presence of minimal electromagnetic interactions, all the subsidiary conditions which the spin 2 field should satisfy and it is only the minimal electromagnetic interaction which is consistent.

We have also found that there exists not only one Lagrangian but a one-parameter family of Lagrangians all of which are equally suitable for the description of the spin 2 field.* Corresponding to the different values of the parameter A in the Lagrangian, there exists a class of propagators for the spin 2 field; but the covariant commutation relations for the free field are not dependent on the parameter A.

Finally, it has been shown that a consistent quantisation of the spin 2 field in the presence of electromagnetic interactions is not possible; the equal time commutator between a field variable and its hermitian conjugate, which is zero in the absence of any interaction, does not vanish when the electromagnetic interaction is switched on.

2.1. Lagrangian

The spin 2 field is defined by the following set of equations:

$$(\Box - m^2)\psi_{\mu\nu} = 0 \,, \tag{2.1}$$

$$\partial_\mu \psi_{\mu\nu} = 0 \,, \tag{2.2}$$

$$\psi_{\mu\mu} = 0 \,, \tag{2.3}$$

---

* After this work has been completed, Dr. S. Kamefuchi has brought it to the notice of the author that it was pointed out implicitly by Fronsdal[7] also that there should exist a two-parameter family of Lagrangians for the description of the spin 2 field. In any case, our approach is different from that of Fronsdal.

where $\psi_{\mu\nu}$ is a symmetric tensor of rank 2. We shall show that there exists a one-parameter family of Lagrangians from which all the equations (2.1), (2.2) and (2.3) can be derived with the help of a variational principle. Let $\mathcal{L}$ be the Lagrangian density of the system,

$$\begin{aligned}\mathcal{L} = & -\frac{\partial \psi_{\mu\nu}^*}{\partial x^\lambda}\cdot\frac{\partial \psi_{\mu\nu}}{\partial x^\lambda} - m^2\psi_{\mu\nu}^*\psi_{\mu\nu} + \frac{\partial \psi_{\mu\nu}^*}{\partial x^\lambda}\cdot\frac{\partial \psi_{\lambda\nu}}{\partial x^\mu} \\ & + \frac{\partial \psi_{\mu\nu}^*}{\partial x^\mu}\cdot\frac{\partial \psi_{\lambda\nu}}{\partial x^\lambda} + A\left(\frac{\partial \psi_{\mu\nu}^*}{\partial x^\mu}\cdot\frac{\partial \psi}{\partial x^\nu} + \frac{\partial \psi^*}{\partial x^\nu}\cdot\frac{\partial \psi_{\mu\nu}}{\partial x^\mu}\right) \\ & + B\,\frac{\partial \psi^*}{\partial x^\nu}\cdot\frac{\partial \psi}{\partial x^\nu} + m^2 C\,\psi^*\psi \quad ,\end{aligned} \tag{2.4}$$

where $\psi = \psi_{\mu\mu}$ and A, B, C are arbitrary constants. The equation of motion that follows from the Lagrangian (2.4) is

$$\begin{aligned}(\Box - m^2)\psi_{\mu\nu} & - \left(\frac{\partial^2 \psi_{\lambda\nu}}{\partial x^\lambda \partial x^\mu} + \frac{\partial^2 \psi_{\lambda\mu}}{\partial x^\lambda \partial x^\nu}\right) - A\frac{\partial^2 \psi}{\partial x^\mu \partial x^\nu} \\ & - \delta_{\mu\nu}\left[A\frac{\partial^2 \psi_{\rho\lambda}}{\partial x^\rho \partial x^\lambda} + B\Box\psi\right] + \delta_{\mu\nu}m^2 C\psi = 0 .\end{aligned} \tag{2.5}$$

By putting $\mu = \nu$ in equation (2.5) and summing over μ we get

$$(4A+2)\frac{\partial^2 \psi_{\rho\lambda}}{\partial x^\rho \partial x^\lambda} + (4B+A-1)\Box\psi + (1-4C)m^2\psi = 0 . \tag{2.6}$$

Now differentiating equation (2.5) with respect to x_μ and equation (2.6) with respect to x_ν and adding the resultant equations one obtains

$$\left[(4B+A-1)(A+1)(4A+2)^{-1}-(B+A)\right]\Box\frac{\partial\psi}{\partial x^\nu}$$
$$+m^2\left[(1-4C)(A+1)(4A+2)^{-1}+C\right]\frac{\partial\psi}{\partial x^\nu}-m^2\frac{\partial\psi_{\mu\nu}}{\partial x^\mu}=0\,.$$

(2.7)

In equation (2.7) $A\neq-\frac{1}{2}$.

If we put $B=(\frac{3}{2}A^2+A+\frac{1}{2})$, then equation (2.7) reduces to

$$\frac{\partial\psi_{\mu\nu}}{\partial x^\mu}-\left[(1-4C)(A+1)(4A+2)^{-1}+C\right]\frac{\partial\psi}{\partial x^\nu}=0\,,$$

(2.8)

when $m\neq 0$.

Again differentiating equation (2.8) with respect to and subtracting the resultant equation from equation (2.6) we get

$$\left[(4B+A-1)(4A+2)^{-1}+\left\{(1-4C)(A+1)(4A+2)^{-1}+C\right\}\right]\Box\psi$$
$$+(1-4C)(4A+2)^{-1}m^2\psi=0\,.$$

(2.9)

If $C=3A^2+3A+1$ and $m\neq 0$, then equation (2.9) gives

$$\psi=0\,.$$

(2.10)

Now from (2.8) and (2.10) follows the other subsidiary condition:

$$\frac{\partial \psi_{\mu\nu}}{\partial x^\mu} = 0 \quad , \tag{2.11}$$

and also by putting equation (2.10) and (2.11) in equation (2.5) one obtains the Klein-Gordan equation for the spin 2 field.

Finally, the Lagrangian written with only one arbitrary parameter is

$$\begin{aligned}\mathcal{L} = &-\frac{\partial \psi_{\mu\nu}^*}{\partial x^\lambda}\cdot\frac{\partial \psi_{\mu\nu}}{\partial x^\lambda} - m^2 \psi_{\mu\nu}^* \psi_{\mu\nu} + \frac{\partial \psi_{\mu\nu}^*}{\partial x^\lambda}\cdot\frac{\partial \psi_{\lambda\nu}}{\partial x^\mu} \\ &+ \frac{\partial \psi_{\mu\nu}^*}{\partial x^\mu}\cdot\frac{\partial \psi_{\lambda\nu}}{\partial x^\lambda} + A\left(\frac{\partial \psi_{\mu\nu}^*}{\partial x^\mu}\cdot\frac{\partial \psi}{\partial x^\nu} + \frac{\partial \psi^*}{\partial x^\nu}\cdot\frac{\partial \psi_{\mu\nu}}{\partial x^\mu}\right) \\ &+ \left(\tfrac{3}{2}A^2 + A + \tfrac{1}{2}\right)\frac{\partial \psi^*}{\partial x^\mu}\cdot\frac{\partial \psi}{\partial x^\mu} + m^2\left(3A^2 + 3A + 1\right)\psi^*\psi \quad ,\end{aligned} \tag{2.12}$$

where $A \neq -\frac{1}{2}$.

If we put $A = -1$ in the above Lagrangian and introduce a symmetric traceless tensor field $\varphi_{\mu\nu}$ defined by the relation

$$\psi_{\mu\nu} = \varphi_{\mu\nu} + \tfrac{1}{4}\delta_{\mu\nu}\psi_{\lambda\lambda} \quad , \tag{2.13}$$

and identify the field $\psi_{\lambda\lambda}$ with a scalar field C, then the Lagrangian (2.12) reduces to that obtained by Fierz and Pauli,

$$\begin{aligned}&\mathcal{L}\left(\psi_{\mu\nu}, \psi_{\mu\nu,\lambda}\right) = \mathcal{L}\left(\varphi_{\mu\nu}, \varphi_{\mu\nu,\lambda} \,;\, C, C_{,\lambda}\right) \\ &= -\frac{\partial \varphi_{\mu\nu}^*}{\partial x^\lambda}\cdot\frac{\partial \varphi_{\mu\nu}}{\partial x^\lambda} - m^2 \varphi_{\mu\nu}^* \varphi_{\mu\nu} + \tfrac{3}{4} m^2 C^* C \\ &+ 2\frac{\partial \varphi_{\mu\nu}}{\partial x^\mu}\cdot\frac{\partial \varphi_{\lambda\nu}}{\partial x^\lambda} + \tfrac{3}{8}\frac{\partial C^*}{\partial x^\lambda}\cdot\frac{\partial C}{\partial x^\lambda} - \tfrac{1}{2}\left(\frac{\partial \varphi_{\mu\nu}^*}{\partial x^\mu}\cdot\frac{\partial C}{\partial x^\nu} \right. \\ &\qquad\qquad \left. + \frac{\partial C^*}{\partial x^\mu}\cdot\frac{\partial \varphi_{\mu\nu}}{\partial x^\nu}\right)\end{aligned} \tag{2.14}$$

If the field $\psi_{\mu\nu}$ undergoes the transformation

$$\psi_{\mu\nu} \rightarrow \psi'_{\mu\nu} = \psi_{\mu\nu} + a\,\delta_{\mu\nu}\psi \tag{2.15}$$

where a is an arbitrary constant, then the Lagrangian (2.12) and the field equations (2.5) remain invariant provided the constant A is also transformed in the following way:

$$A \rightarrow A' = 2a + A + 4aA \,. \tag{2.16}$$

As the different values of the parameter A determine the proportion in which the scalar field ψ is mixed up with the other fields and as the Lagrangian is invariant under the transformations (2.15) and (2.16), the dynamical variables such as the energy, momentum will not depend on the values of A.

3. Electromagnetic Interactions

The minimal electromagnetic interaction is introduced in the Lagrangian (2.12) by the gauge invariant replacement, $\partial_\mu \rightarrow \Pi_\mu = \partial_\mu - ie\varphi_\mu$, when ∂_μ operates on $\psi_{\mu\nu}$ or $\partial_\mu \rightarrow \Pi^*_\mu = \partial_\mu + ie\varphi_\mu$ when ∂_μ operates on $\psi^*_{\mu\nu}$. Here φ_μ is the electromagnetic potential.

The dynamic equation is given by

$$\Pi^2\psi_{\mu\nu} - \frac{1}{2}\left(\Pi_\mu\Pi_\lambda\psi_{\lambda\nu} + \Pi_\nu\Pi_\lambda\psi_{\lambda\mu}\right) - \frac{1}{2}\left(\Pi_\lambda\Pi_\mu\psi_{\lambda\nu} + \Pi_\lambda\Pi_\nu\psi_{\lambda\mu}\right)$$
$$- A\,\Pi_\mu\Pi_\nu\psi - \frac{1}{2}ieA f_{\mu\nu}\psi - \delta_{\mu\nu}\left[A\,\Pi_\rho\Pi_\lambda\psi_{\rho\lambda} + \left(\tfrac{3}{2}A^2 + A + \tfrac{1}{2}\right)\Pi^2\psi\right]$$
$$- m^2\left[\psi_{\mu\nu} - \delta_{\mu\nu}\left(3A^2 + 3A + 1\right)\psi\right] = 0 \,, \tag{3.1}$$

where

$$f_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu \,,$$
$$\Pi_\mu\Pi_\nu - \Pi_\nu\Pi_\mu = -ie f_{\mu\nu} \,,$$
$$\Pi_\mu^*\Pi_\nu^* - \Pi_\nu^*\Pi_\mu^* = ie f_{\mu\nu} \,.$$

Also by putting $\mu = \nu$ in equation (3.1) and summing over μ one obtains

$$\Pi_\rho\Pi_\lambda\psi_{\rho\lambda} + \frac{1}{2}(3A+1)\Pi^2\psi - \frac{3}{2}(2A+1)m^2\psi = 0 \,, \tag{3.2}$$

where $A \neq -\frac{1}{2}$.

After the introduction of the electromagnetic interaction it is no more possible to deduce the subsidiary conditions in the form of equations (2.10) and (2.11). Nevertheless, one can obtain ten equations containing only the first derivative with respect to time of the field $\psi_{\mu\nu}$ and this shows that the field has no more degrees of freedom than are necessary to describe particles with spin 2. Three of the ten subsidiary equations are obtained from equation (3.1) by putting $\mu = i$, $\nu = 4$,

$$\Pi_k^2 \psi_{i4} - \Pi_i \Pi_\lambda \psi_{\lambda 4} - \Pi_4 \Pi_k \psi_{ki} - A \Pi_i \Pi_4 \psi - \tfrac{1}{2} ie A f_{i4} \psi$$
$$- m^2 \psi_{i4} + \tfrac{1}{2} ie f_{\rho i} \psi_{\rho 4} + \tfrac{1}{2} ie f_{\rho 4} \psi_{\rho i} = 0$$

(3.3)

where $i, k = 1, 2, 3$.

The fourth subsidiary condition is obtained by adding the $(4,4)$ component of equation (1) to equation (2) ,

$$\Pi_k^2 \psi_{44} + \Pi_k \Pi_i \psi_{ki} + A \Pi_k^2 \psi - \tfrac{1}{2} m^2 (3A+1) \psi$$
$$- m^2 \psi_{44} = 0 .$$

(3.4)

Applying the operators Π_μ and $(A+1)\Pi_\nu$, respectively, to equations (3.1) and (3.2), and adding the resultant equations we get

$$- m^2 \left[\Pi_\mu \psi_{\mu\nu} + \tfrac{1}{2} (3A+1) \Pi_\nu \psi \right] - 2ie f_{\mu\lambda} \Pi_\lambda \psi_{\mu\nu}$$
$$- ie \Pi_\lambda f_{\mu\lambda} \psi_{\mu\nu} + ie f_{\mu\nu} \Pi_\lambda \psi_{\lambda\mu} + 2ie A f_{\lambda\nu} \Pi_\lambda \psi$$
$$+ ie A \Pi_\lambda f_{\lambda\nu} \psi - \tfrac{1}{2} ie A \Pi_\mu (f_{\mu\nu} \psi) + \tfrac{1}{2} ie \Pi_\mu (f_{\rho\mu} \psi_{\rho\nu})$$
$$+ \tfrac{1}{2} ie \Pi_\mu (f_{\rho\nu} \psi_{\rho\mu}) = 0 .$$

(3.5)

Equation (3.5) gives four more subsidiary conditions.

Again applying the operator π_ν to equation (3.5) and comparing it with equation (3.2) another subsidiary condition is obtained:

$$
\begin{aligned}
&-\tfrac{3}{2} m^4 (2A+1)\psi - ie\,\pi_\nu f_{\mu\lambda}\pi_\lambda \psi_{\mu\nu} \\
&+ ie\,\pi_\nu f_{\mu\nu}\pi_\lambda \psi_{\lambda\mu} + 2e^2 f_{\mu\nu} f_{\rho\mu}\psi_{\rho\nu} \\
&+ ie\,\pi_\nu f_{\lambda\nu}\pi_\lambda \psi - \tfrac{5}{4} e^2 A f_{\nu\lambda} f_{\nu\lambda}\psi = 0 .
\end{aligned}
\tag{3.6}
$$

The tenth subsidiary condition is obtained by differentiating equation (3.4) with respect to time.

4. Quantisation of Free Spin 2 Field.

4.1. Equal Time Commutation Relations

We define a field ψ_{ik}^{2} given by

$$\psi_{ik}^{2} = C_{ikmn}\psi_{mn} \tag{4.1}$$

where $C_{ikmn} = \frac{1}{2}\left(\delta_{im}\delta_{kn} + \delta_{in}\delta_{km} - \frac{2}{3}\delta_{ik}\delta_{mn}\right)$

The field ψ_{ik}^{2} has five independent components and is indeed characterised by spin 2 for three dimensional rotations. In the canonical formalism the fields ψ_{i4} , ψ_{ll} , ψ_{44} are treated as dependent 'coordinates' and can be expressed in terms of the canonically independent variables and their conjugate momenta with the aid of the following equations:*

$$\left(-\tfrac{2}{3}\partial^2 + m^2\right)\Psi_{ll} = -\partial_k\partial_i \Psi_{ki}^{2} \quad , \tag{4.2}$$

$$\left(m^2 - \tfrac{1}{2}\partial^2\right)\Psi_{k4} = i\left(\delta_{kn} - \tfrac{1}{2}\partial_k\partial_n/m^2\right)\partial_l \Pi_{ln}^{+} \quad , \tag{4.3}$$

$$\Psi_{44} = -\Psi_{ll} \tag{4.4}$$

where Π_{ln}^{+} is the momentum conjugate to the field variable Ψ_{ln}^{2+} .

The momenta Π_{k4} , Π_{ll} and Π_{44} conjugate, respectively, to the fields Ψ_{k4} , Ψ_{ll} and Ψ_{44} are given by

$$\Pi_{k4} = -\tfrac{1}{2}i\left(\partial_i \Psi_{ik}^{2+} - \tfrac{2}{3}\partial_k \Psi_{ll}^{+}\right) \quad , \tag{4.5}$$

$$\Pi_{ll} = 0 \quad , \tag{4.6}$$

$$\Pi_{44} = 0 \quad . \tag{4.7}$$

By using the constraint equations (4.2), (4.3), (4.4) and (4.5) one obtains the following operator generator for an infinitesimal variation of $\Psi_{\mu\nu}$ on a space-like surface σ :

---

* In most of the following we choose for convenience A = -1 .

$$G = \int \Pi^{\ell n} D_{\ell n k r}\, \delta \psi^{2}_{kr}\, d\sigma \ , \tag{4.8}$$

where

$$D_{\ell n k r} = \Big[\delta_{\ell k} \delta_{n r} + \tfrac{1}{2} \left(A_{\ell k r n} + A_{\ell r k n} \right) - \tfrac{1}{3} \delta_{k r} \partial_\ell \partial_n \left(\mu^2 - \tfrac{2}{3} \partial^2 \right)^{-1} \Big]$$

and $A_{\ell k r n}$ is given by

$$A_{\ell k r n} = \partial_\ell \partial_k \left(m^2 \delta_{r n} + \tfrac{1}{6} \partial_r \partial_n - \tfrac{2}{3} \partial^2 \delta_{r n} \right) \Big\{ (m^2 - \tfrac{1}{2} \partial^2) \cdot (m^2 - \tfrac{2}{3} \partial^2) \Big\}^{-1}$$

The commutation relations may now be derived with the aid of the generator (4.8)[9]. Thus, we find

$$\left[\psi^{2}_{im}(x), \Pi^{2}_{\ell n}(y) \right] = i F_{im\ell n}\, \delta(x-y) \ , \tag{4.9}$$

where

$$F_{im\ell n} = \Big(C_{im\ell n} + \tfrac{1}{3} \delta_{\ell n} \partial_i \partial_m / m^2 + \tfrac{1}{3} \delta_{im} \partial_\ell \partial_n / m^2 - \tfrac{1}{4} \delta_{ni} \partial_\ell \partial_m / m^2 - \tfrac{1}{4} \delta_{nm} \partial_\ell \partial_i / m^2 - \tfrac{1}{4} \delta_{\ell i} \partial_n \partial_m / m^2 - \tfrac{1}{4} \delta_{\ell m} \partial_i \partial_n / m^2 - \tfrac{1}{9} \delta_{\ell n} \delta_{im} \partial^2 / m^2 \Big) . \tag{4.9a}$$

The commutation relations for the kinematically dependent components of the field are given by

$$\left[\psi_{im}^{2}(x), \psi_{k4}^{+}(y)\right] = \Big\{\tfrac{1}{2}\delta_{km}\partial_i/m^2 + \tfrac{1}{2}\delta_{ik}\partial_m/m^2 - \tfrac{1}{3}\delta_{im}\partial_k/m^2 + \tfrac{2}{9}\delta_{im}\partial_k\partial^2/m^4 - \tfrac{2}{3}\partial_k\partial_i\partial_m/m^4\Big\}\delta(x-y), \tag{4.10}$$

$$\left[\psi(x), \pi_{ln}^{2}(y)\right] = i/m^2\left(\tfrac{1}{3}\delta_{ln}\partial^2 - \partial_l\partial_n\right)\delta(x-y), \tag{4.11}$$

$$\left[\psi(x), \psi_{k4}^{+}(y)\right] = -\frac{2}{3m^4}\partial^2\partial_k\,\delta(x-y), \tag{4.12}$$

$$\left[\pi_{m4}(x), \psi_{k4}(y)\right] = \tfrac{1}{4}i/m^2\Big\{\delta_{km}\partial^2 + \tfrac{1}{3}\partial_k\partial_m\Big\}\delta(x-y), \tag{4.13}$$

$$\left[\psi^{+}(x), \psi(y)\right] = 0, \tag{4.14}$$

$$\left[\psi_{im}^{+}(x), \psi_{kl}(y)\right] = 0, \tag{4.14a}$$

$$\left[\pi_{im}^{+}(x), \pi_{kl}(y)\right] = 0, \tag{4.14b}$$

$$\left[\pi_{k4}^{+}(x), \pi_{l4}(y)\right] = 0, \tag{4.14c}$$

$$\left[\psi_{k4}^{+}(x), \psi_{l4}(y)\right] = 0. \tag{4.14d}$$

It may be noted that in spite of the non-local character of the constraint equations, all the components of the field satisfy local commutation relations.

4.2. The Covariant Commutation Relations and the Propagators

The wave equation (2.5) can be written in the form:

$$\Lambda_{\mu\nu\rho\lambda}\Psi_{\rho\lambda} = 0 \,, \tag{4.15}$$

where

$$\begin{aligned}\Lambda_{\mu\nu\rho\lambda} = \Big[&(\Box - m^2)\delta_{\mu\lambda}\delta_{\rho\nu} - (\partial_\lambda\partial_\mu\delta_{\rho\nu} + \partial_\lambda\partial_\nu\delta_{\rho\mu}) \\ &- A\delta_{\lambda\rho}\partial_\mu\partial_\nu - A\partial_\lambda\partial_\rho\delta_{\mu\nu} - B\delta_{\mu\nu}\delta_{\lambda\rho}\Box + m^2 D\delta_{\lambda\rho}\delta_{\mu\nu}\Big], \\ B = &\left(\tfrac{3}{2}A^2 + A + \tfrac{1}{2}\right), \quad D = (3A^2 + 3A + 1) \,.\end{aligned} \tag{4.15a}$$

As the wave function $\psi_{\rho\lambda}$ should satisfy the Klein-Gordan equation, there exists an inverse operator $d_{\mu'\nu'\rho\lambda}$ which obeys the relation

$$d_{\mu'\nu'\rho\lambda}\Lambda_{\mu\nu\rho\lambda} = (\Box - m^2)(\delta_{\mu'\mu}\delta_{\nu'\nu} + \delta_{\mu'\nu}\delta_{\nu'\mu}) \,. \tag{4.16}$$

The operator $d_{\mu'\nu'\rho\lambda}$ is a function of the derivation operators ∂_μ and the highest order of such operators appearing in $d_{\mu'\nu'\rho\lambda}$ is fourth as the maximum value of the spin of the particles described by the wave function is 2. Let the operator $d_{\mu'\nu'\rho\lambda}$ be given by

$$
\begin{aligned}
d_{\mu'\nu'\rho\lambda} = \Big(& a\,\delta_{\mu'\nu'\rho\lambda} + b\,\delta_{\mu'\rho}\delta_{\nu'\lambda} + c\,\delta_{\mu'\lambda}\delta_{\nu'\rho} \\
& + d\,\delta_{\mu'\nu'}\partial_\rho\partial_\lambda + e\,\delta_{\mu'\rho}\partial_{\nu'}\partial_\lambda + f\,\delta_{\mu'\lambda}\partial_{\nu'}\partial_\rho \\
& + g\,\delta_{\nu'\rho}\partial_{\mu'}\partial_\lambda + h\,\delta_{\nu'\lambda}\partial_{\mu'}\partial_\rho + i\,\delta_{\rho\lambda}\partial_{\mu'}\partial_{\nu'} \\
& + j\,\Box\,\delta_{\mu'\nu'}\delta_{\rho\lambda} + k\,\Box\,\delta_{\mu'\rho}\delta_{\nu'\lambda} + l\,\Box\,\delta_{\mu'\lambda}\delta_{\nu'\rho} \\
& + m\,\partial_{\mu'}\partial_{\nu'}\partial_\rho\partial_\lambda + n\,\Box\,\delta_{\mu'\nu'}\partial_\rho\partial_\lambda + o\,\Box\,\delta_{\mu'\rho}\partial_{\nu'}\partial_\lambda \\
& + p\,\Box\,\delta_{\mu'\lambda}\partial_{\nu'}\partial_\rho + q\,\Box\,\delta_{\nu'\rho}\partial_{\mu'}\partial_\lambda + r\,\Box\,\delta_{\nu'\lambda}\partial_{\mu'}\partial_\rho \\
& + s\,\Box\,\delta_{\rho\lambda}\partial_{\mu'}\partial_{\nu'} + t\,\Box^2\delta_{\mu'\nu'}\delta_{\rho\lambda} + u\,\Box^2\delta_{\mu'\rho}\delta_{\nu'\lambda} \\
& + v\,\Box^2\delta_{\mu'\lambda}\delta_{\nu'\rho} \Big)
\end{aligned}
\tag{4.17}
$$

Now using the values of the operators $d_{\mu'\nu'\rho\lambda}$ and $\Lambda_{\mu\nu\rho\lambda}$ given, respectively, by equations (4.15a) and (4.17) one obtains from equation (4.16)

$$
\begin{aligned}
d_{\mu\nu\rho\lambda} = \Big[& \delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho} - \tfrac{2}{3}\delta_{\mu\nu}\delta_{\rho\lambda} \\
& - \delta_{\mu\rho}\partial_\nu\partial_\lambda/m^2 - \delta_{\mu\lambda}\partial_\nu\partial_\rho/m^2 \\
& - \delta_{\nu\rho}\partial_\mu\partial_\lambda/m^2 - \delta_{\nu\lambda}\partial_\mu\partial_\rho/m^2
\end{aligned}
$$

$$+ \tfrac{2}{3}\delta_{\mu\nu}\partial_\rho\partial_\lambda/m^2 + \tfrac{2}{3}\delta_{\rho\lambda}\partial_\mu\partial_\nu/m^2 + \tfrac{4}{3}\partial_\mu\partial_\nu\partial_\rho\partial_\lambda/m^4$$

$$- \tfrac{2}{3}(A+1)(2A+1)^{-1} m^{-4}\Big[\delta_{\mu\nu}\partial_\rho\partial_\lambda + \delta_{\rho\lambda}\partial_\mu\partial_\nu$$

$$+ m^2 A(2A+1)^{-1}\delta_{\mu\nu}\delta_{\rho\lambda} - \tfrac{1}{2}(A+1)(2A+1)^{-1}\Box\,\delta_{\mu\nu}\delta_{\rho\lambda}\Big](\Box - m^2). \tag{4.18}$$

The commutation relations and the expressions for the propagators are given by[10]

$$\left[\psi_{\mu\nu}(x), \psi^{+}_{\rho\lambda}(y)\right] = d_{\mu\nu\rho\lambda}\,\Delta(x-y), \tag{4.19}$$

$$\langle 0|\,T\left(\psi_{\mu\nu}(x)\,\psi^{+}_{\rho\lambda}(y)\right)|0\rangle = d_{\mu\nu\rho\lambda}\,\Delta_F(x-y), \tag{4.20}$$

where

$$\Delta(x-y) = -i(8\pi^3)^{-1}\int e^{ik(x-y)}\,\delta(k^2+m^2)\,\epsilon(k^0)\,d^4k,$$

$$\Delta_F(x-y) = -i(2\pi)^{-4}\int e^{ik(x-y)}\,(k^2+m^2-i\epsilon)^{-1}\,d^4k,$$

$$k^2 = k_\lambda k_\lambda, \quad d^4k = d^3\underset{\sim}{k}\,(-i\,dk_4).$$

As $(\Box - m^2)\Delta(x-y) = 0$, the commutation relations are not dependent on A.

5.1. Quantisation of the Spin 2 Field in the Presence of the Electromagnetic Interaction

Let us assume that it is possible to quantise consistently the spin 2 field in interaction with the electromagnetic field and the following are the commutation relations on a space-like surface:

$$\left[\psi^2_{im}(x), \Pi_{ln}(y)\right] = i\left\{F_{imln} + G_{imln}\right\}\delta(x-y), \tag{5.1}$$

$$\left[\psi^{2+}_{im}(x), \Pi^+_{ln}(y)\right] = i\left\{F_{imln} + G^*_{imln}\right\}\delta(x-y), \tag{5.2}$$

$$\left[\psi^2_{im}(x), \psi^{2+}_{ln}(y)\right] = 0, \tag{5.3}$$

$$\left[\Pi_{im}(x), \Pi^+_{ln}(y)\right] = A_{imln}\,\delta(x-y), \tag{5.4}$$

$$\left[\psi(x), \Pi_{ln}(y)\right] = i/m^2\left\{\left(\tfrac{1}{3}\delta_{ln}\partial^2 - \partial_l\partial_n\right) + B_{ln}\right\}\delta(x-y), \tag{5.5}$$

$$\left[\psi^+(x), \Pi^+_{ln}(y)\right] = i/m^2\left\{\left(\tfrac{1}{3}\delta_{ln}\partial^2 - \partial_l\partial_n\right) + B^*_{ln}\right\}\delta(x-y), \tag{5.6}$$

where

$$i\Pi_{im} = \left\{-\Pi_4^*\psi^+_{im} + \tfrac{1}{2}\Pi_i^*\psi^+_{4m} + \tfrac{1}{2}\Pi_m^*\psi^+_{4i} - \Pi_l^*\psi^+_{l4}\delta_{im} + \Pi_4^*\psi^+_{ll}\delta_{im}\right\}, \tag{5.7}$$

$$i\Pi^{+}_{im} = \Big\{ -\Pi_4 \Psi_{im} + \tfrac{1}{2}\Pi_i \Psi_{4m} + \tfrac{1}{2}\Pi_m \Psi_{4i} - \Pi_l \Psi_{l4}\delta_{im} + \Pi_4 \Psi_{ll}\delta_{im} \Big\} . \qquad (5.8)$$

The momenta Π_{im} and Π^{+}_{im} reduce, respectively, to Π^{2}_{im} and Π^{2+}_{im} when the interaction is switched off. The quantities G_{imln} , A_{imln} and B_{ln} are functions of $\underline{\varphi}$ and f_{ik} , so that they vanish when there is no electromagnetic interaction and the commutation relations (5.1) - (5.6) reduce to the free field commutation relations.

In the presence of the electromagnetic interaction the equations which are analogous to equations (4.2) and (4.3) are, respectively,

$$\left(-\tfrac{2}{3}\Pi^2 + m^2\right)\Psi_{ll} = -\Pi_k \Pi_i \Psi^{2}_{ki} \qquad (5.9)$$

and

$$i\Pi_k \Pi^{+}_{ki} = A_{ik}\Psi_{k4} + \tfrac{1}{2} ie f_{k4}\Psi^{2}_{ki} - \tfrac{1}{3} ie f_{i4}\Psi_{ll} , \qquad (5.10)$$

where

$$A_{ik} = \left(m^2\delta_{ik} + \tfrac{1}{2}\Pi_i\Pi_k - \tfrac{1}{2}\delta_{ik}\Pi^2 - ie f_{ki}\right) .$$

Similarly,

$$\left(-\tfrac{2}{3}\Pi^{*2} + m^2\right)\Psi^{+}_{ll} = -\Pi^{*}_k \Pi^{*}_i \Psi^{2+}_{ki} , \qquad (5.11)$$

$$i\Pi^{*}_k \Pi_{ki} = A^{*}_{ik}\Psi^{+}_{k4} - \tfrac{1}{2} ie f_{k4}\Psi^{2+}_{ik} + \tfrac{1}{3} ie f_{i4}\Psi^{+}_{ll} , \qquad (5.12)$$

where

$$A_{ik}^{*} = \left(m^2 \delta_{ik} + \tfrac{1}{2}\Pi_i^{*}\Pi_k^{*} - \tfrac{1}{2}\delta_{ik}\Pi^{*2} + ie f_{ki}\right).$$

So, we get the following commutation relations:

$$\left[\psi_{ll}^{+}(x), \psi_{nn}(y)\right] = 0 , \tag{5.13}$$

$$\left[\psi_{ll}^{+}(x), \psi_{im}^{2}(y)\right] = 0 \tag{5.14}$$

and

$$\left[A_{ik}^{*}(x)\,\psi_{k4}^{*}(x) , A_{nl}(y)\,\psi_{l4}(y)\right]$$

$$= -\Pi_k^{*}(x)\,\Pi_l(y)\left[\Pi_{ki}(x), \Pi_{ln}^{+}(y)\right]$$

$$+ \tfrac{1}{2} e f_{s4}(y)\,\Pi_k^{*}(x)\left[\Pi_{ki}(x), \psi_{sn}^{2}(y)\right]$$

$$+ \tfrac{1}{2} e f_{s4}(x)\,\Pi_k(y)\left[\Pi_{kn}^{+}(y), \psi_{si}^{2+}(x)\right]$$

$$+ \tfrac{1}{3} e f_{i4}(x)\,\Pi_k(y)\left[\psi^{+}(x), \Pi_{kn}^{+}(y)\right]$$

$$+ \tfrac{1}{3} e f_{n4}(y)\,\Pi_k^{+}(x)\left[\psi(y), \Pi_{ki}(x)\right]$$

$$\neq 0 .$$

(5.15)

Equation (5.15) shows that the operators ψ_{i4}^{+} and ψ_{k4} do not commute on a space-like surface.

5.2. Equal Time Commutation Relations between Two Field Variables in the Presence of Interactions.

In this section we shall show that if the free field commutator between two operators vanishes on a space-like surface, then it should vanish on a space-like surface in the presence of interaction also.

Let us consider a set of free field operators which satisfy the wave equation

$$\Lambda_{\alpha\beta}\, Q_\beta(x) = 0 \quad , \tag{5.16}$$

where $\Lambda_{\alpha\beta}$ is a function of the derivation operators $\partial_\mu(x)$. The covariant commutation relations between the operators $Q_\alpha(x)$ are given by

$$\left[Q_\alpha^+(x), Q_\beta(y) \right] = i d_{\alpha\beta}\, \Delta(x-y) \tag{5.17}$$

where

$$\Lambda_{\alpha\beta}\, d_{\beta\gamma} = (\Box - m^2)\, \delta_{\alpha\gamma} \ ,$$

m is the mass of the particle described by the wave functions $Q_\alpha(x)$.

In general the operators $Q_\alpha(x)$ are made up of two sets of field quantities: Q_1 , Q_2 , Q_3 ,......, Q_n , the canonically independent coordinates and Q_{n+1} , Q_{n+2} ,...... ..., Q_r , the dependent variables which can always be expressed in terms of the canonically independent field variables and their conjugate momenta. So the equal time commutation relations between the canonically independent components are

$$\left[Q_\alpha^+(\underset{\sim}{x},t) \, , \, Q_\beta(\underset{\sim}{y},t) \right] = 0 \tag{5.18}$$

where

$$\alpha, \beta = 1, 2, 3, \cdots, n.$$

It may so happen that the equal time commutation relations between some of the dependent variables also vanish.

$$\left[Q_\gamma^+(\underset{\sim}{x},t) \, , \, Q_s(\underset{\sim}{y},t) \right] = 0 \, , \tag{5.19}$$

where

$Q_\gamma^+(\underset{\sim}{x},t)$ and $Q_s(\underset{\sim}{y},t)$ are dependent coordinates.

Now we shall show that the commutation relations (5.18) and (5.19) are true in the presence of interaction also. The equation of motion for the interacting fields $\underline{Q}_\alpha(x)$ in the Heisenberg picture are

$$\Lambda_{\alpha\beta}(\partial) \, \underline{Q}_\beta(x) = \underline{J}_\alpha(x) \, , \tag{5.20}$$

where

$$\underline{J}_\alpha(x) = \frac{\partial \mathcal{L}_1(x)}{\partial \underline{Q}_\alpha(x)} - \partial_\mu \left(\frac{\partial \mathcal{L}_1(x)}{\partial \underline{Q}_{\alpha,\mu}} \right)$$

and $\mathcal{L}_1(x)$ is the interaction Lagrangian density.

By integrating the equation (5.20) we get[11]

$$\underline{Q}_\alpha(x) = Q_\alpha^{in}(x) + \int d_{\alpha\beta}(\partial)\,\Delta^{ret.}(x-x')\,\underline{J}_\beta(x')\,dx' \,, \tag{5.21}$$

where

$$\underline{Q}_\alpha(x) = Q_\alpha^{in}(x) \quad \text{as } t \to -\infty \,,$$

$$\Delta^{ret.}(x-x') = \frac{1+\epsilon(x-x')}{2}\,\Delta(x-x')$$

and

$d_{\alpha\beta}(\partial)$ is given by equation (5.17).

The operators $Q_\alpha^{in}(x)$ satisfy the equation of motion (5.16) and the free field commutation relations (5.17), (5.18) and (5.19). The canonically independent components of the in-operators $Q_\alpha^{in}(x)$ are related with the Heisenberg operators $\underline{Q}_\alpha(x)$ by the following unitary transformation:

$$U(t)\,\underline{Q}_\alpha(\underset{\sim}{x},t)\,U^{-1}(t) = Q_\alpha^{in}(\underset{\sim}{x},t) \,, \tag{5.22}$$

where $\alpha = 1, 2, 3, \ldots, n$.

The unitary operator $U(t)$ is given by

$$i\,\frac{\partial U(t)}{\partial t} = H_1(t)\,U(t) \,, \tag{5.23}$$

$H_1(t) = \int H_1(\underset{\sim}{x},t)\,d^3\underset{\sim}{x}$, is the interaction Hamiltonia n.

For the dependent components of the field we have the following relations:

$$\mathcal{U}(t)\,\underline{\mathcal{Q}}_r(\underset{\sim}{x},t)\,\mathcal{U}^{-1}(t) = \mathcal{Q}_r^{in}(\underset{\sim}{x},t) + i g_r(\underset{\sim}{x},t) \tag{5.24}$$

where $r = n+1,\ n+2,\ n+3,\ ---$.

and $g_r(\underset{\sim}{x},t)$ is a functional of the fields $\mathcal{Q}_\alpha^{in}(\underset{\sim}{x},t)$.

According to a general theorem proved in ref. (10) the expectation value of the operators $\underline{\mathcal{Q}}_r(\underset{\sim}{x},t)$ or any functional $f(\underline{\mathcal{Q}}_r(\underset{\sim}{x},t))$ of $\underset{\sim}{\mathcal{Q}}_r(\underset{\sim}{x},t)$ is not dependent on the surface term $\underline{g}_r(\underset{\sim}{x},t)$. The rule is that for the calculation of expectation values of $f(\underline{\mathcal{Q}}_r(\underset{\sim}{x},t))$ one can write

$$\left\langle \left| f\left(\underline{\mathcal{Q}}_r(\underset{\sim}{x},t)\right) \right| \right\rangle = \left\langle \left| \mathcal{U}'^{-1}(t)\, f\left(\mathcal{Q}_r^{in}(\underset{\sim}{x},t)\right) \mathcal{U}'(t) \right| \right\rangle, \tag{5.25}$$

where $\mathcal{U}'(t)$ is now given by

$$i\frac{\partial \mathcal{U}'(t)}{\partial t} = -\mathcal{L}_1(t)\,\mathcal{U}'(t) \tag{5.26}$$

and

$\mathcal{L}_1(t) = \int \mathcal{L}_1(\underset{\sim}{x},t)\,d^3\underset{\sim}{x}$, is the interaction Lagrangian,

so we get from equations (5.19) and (5.25)

$$\left\langle \left| \left[\underset{\sim}{Q}_r^{\,+}(\underset{\sim}{x},t),\ \underset{\sim}{Q}_s(\underset{\sim}{y},t) \right] \right| \right\rangle$$

$$= \left\langle \left| U'^{-1}(t) \left[Q_r^{in\,+}(\underset{\sim}{x},t),\ Q_s^{in}(\underset{\sim}{y},t) \right] U'(t) \right| \right\rangle \tag{5.27}$$

The relation (5.27) is true in all the representations which are related to each other by unitary transformations, which means that the commutator itself is zero ,

$$\left[\underset{\sim}{Q}_r^{\,+}(\underset{\sim}{x},t),\ \underset{\sim}{Q}_s(\underset{\sim}{y},t) \right] = 0 \ . \tag{5.28}$$

Equation (4.14d) shows that the free field operators ψ_{k4}^{+} and $\psi_{\ell 4}$ commute on a space-like surface and in section (5.2) we have proved that the commutator between these two operators does not vanish on a space-like surface in the presence of electromagnetic interactions, so this is inconsistent according to the results derived in this section.

5.3. Discussion

In section (5.1) we have assumed that the commutation relations (5.1),....., (5.6) may contain quantities which are functions of the electromagnetic field, but the result obtained there is not dependent on this assumption. As long as equations (5.10) and (5.12) contain both the fields ψ_{ik} (ψ_{ik}^{+}) and its conjugate momenta π_{ik} (π_{ik}^{+}) ; the commutator between the fields ψ_{i4}^{+} and ψ_{k4} would not vanish on a space-like surface. One may try to remove this inconsistency by introducing in the Lagrangian the direct interaction of the electromagnetic field strength with the spin 2 field, but then the theory would be inconsistent in the classical level. It would no more be possible to derive all of the ten subsidiary conditions. This inconsistency in the quantisation of the spin 2 field in interaction with the electromagnetic field demonstrated in Sections (5.1) and (5.2) can be shown to be true for any integral spin field with $s > 1$.

That a consistent quantisation of the higher spin fields with $s > 1$, both integral and half integral, is not possible, was pointed out first by Kusaka and Weinberg in their unpublished work(12). They claimed that the (anti-)commutator between two field operators in the presence of an external electromagnetic field is non-local and does not vanish on a space-like surface. Johnson and Sudarshan(12) have quantised explicitly the spin $^3/_2$ field in interaction with the external electromagnetic field and found that the anticommutator between any two field operators is always local, but these commutation relations are inconsistent with the requirement of the positive definiteness of the anti-commutator between a field operator and its hermitian conjugate.

REFERENCES

1. P.A.M. Dirac, Proc. Roy. Soc., A155, (1936) 447.
2. M. Fierz, Helv. Phys. Acta, 12 (1939) 3.
3. M. Fierz and W. Pauli, Proc. Roy. Soc., A173 (1939) 211.
4. W. Rarita and J. Schwinger, Phys. Rev., 60 (1941) 61.
5. S.N. Gupta, Phys. Rev., 95 (1954) 1334.
6. P.A. Moldauer and K.M. Case, Phys. Rev., 102 (1956) 279.
7. C. Fronsdal, Suppl. Nuovo Cim., 9 (1958) 416.
8. P. Federbush, Nuovo Cim., 19 (1961) 572.
9. J. Schwinger, Phys. Rev., 82 (1951) 919.
10. Y. Takahashi and H. Umezawa, Prog. Theor. Phys., 9 (1953) 1.
11. C.N. Yang and D. Feldman, Phys. Rev., 79 (1950) 972.
12. K. Johnson and E.C.G. Sudarshan, Ann. Phys. 13 (1961) 126.